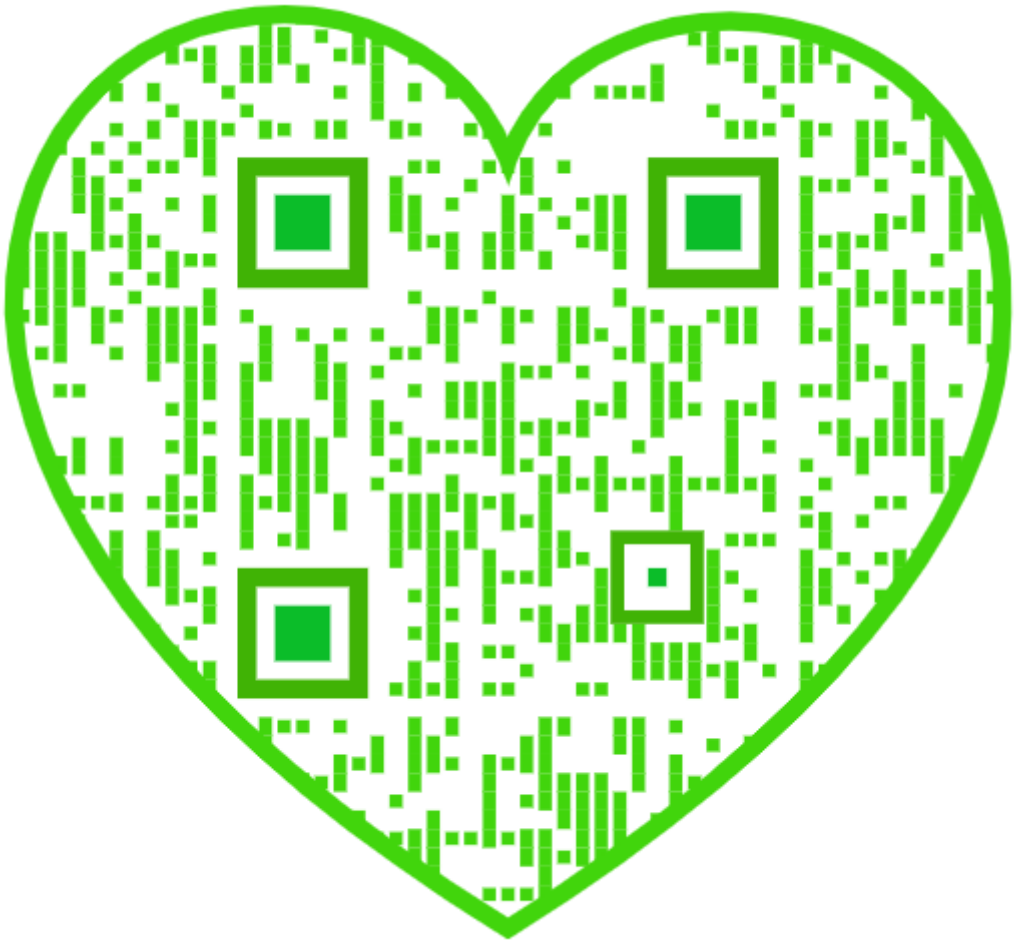


THE

MODEL ELEMENTARY

ARITHMETIC.

ORAL AND WRITTEN.

PART II.

ARRANGED TO SUIT PRESENT GRADATION OF SCHOOL WORK

BY

EDWARD GIDEON, A.M.;

SUPERVISING PRINCIPAL OF THE GEORGE G. MEADE SCHOOL, PHILADELPHIA.

PHILADELPHIA:

ELDREDGE & BROTHER.

1902.

Educ T 119.02.425

HARVARD COLLEGE LIBRARY
BY EXCHANGE

Entered according to Act of Congress, in the year 1901, by
ELDREDGE & BROTHER,
in the Office of the Librarian of Congress, at Washington.

PREFACE.

THIS work presents a course of Arithmetic designed mainly for the pupils of *Elementary Schools*. For this reason the range of topics is limited, but the treatment of each is so full, especially in the fundamental rules and in fractions, as to insure facility and accuracy in performing such computations as are most likely to occur in ordinary business transactions.

This book is designed to follow the Model Elementary Arithmetic, Part I., but it may be used by pupils who have completed any Primary Arithmetic and who are familiar with the four fundamental rules of Arithmetic.

The peculiar features of this, and of the other books of the series, are embraced in the following statements:

First. Forms or models of analysis, correct in expression and logical in reasoning.

Second. Definitions, clear and brief.

Third. In each new subject full solutions of typical examples, combined with proper analytical explanations.

Fourth. Frequent drill exercises to insure accuracy and rapidity.

Fifth. Numerous examples showing the practical application of rules to business and mechanical pursuits.

Sixth. Frequent reviews, in order that pupils may be helped to organize knowledge previously acquired.

CONTENTS

		PAGE
I.	NOTATION AND NUMERATION	5
II.	ADDITION	7
III.	SUBTRACTION	11
IV.	MULTIPLICATION	16
V.	DIVISION	20
VI.	MISCELLANEOUS PROBLEMS	23
VII.	PROPERTIES OF NUMBERS	26
VIII.	FRACTIONS	48
IX.	REDUCTION OF FRACTIONS	58
X.	ADDITION OF FRACTIONS	72
XI.	SUBTRACTION OF FRACTIONS	76
XII.	MULTIPLICATION OF FRACTIONS	82
XIII.	DIVISION OF FRACTIONS	92
XIV.	DECIMAL FRACTIONS	102
XV.	ADDITION OF DECIMALS	109
XVI.	SUBTRACTION OF DECIMALS	111
XVII.	MULTIPLICATION OF DECIMALS	113
XVIII.	DIVISION OF DECIMALS	115
XIX.	REDUCTION OF DECIMALS	125
XX.	DENOMINATE NUMBERS	129
XXI.	REDUCTION OF DENOMINATE NUMBERS	139
XXII.	ADDITION OF COMPOUND NUMBERS	146
XXIII.	SUBTRACTION OF COMPOUND NUMBERS	148
XXIV.	MULTIPLICATION OF COMPOUND NUMBERS	150
XXV.	DIVISION OF COMPOUND NUMBERS	152
XXVI.	MEASUREMENTS	155

EXERCISES IN THE ELEMENTARY RULES.

SECTION I.

NOTATION AND NUMERATION.

1. Express by figures the numbers:—

Six thousand. Seven thousand, three hundred. Nine thousand, one hundred twenty. Three thousand, eight hundred two.

Four thousand, six hundred. Nine thousand, eighty-two. Eight thousand, sixty-four.

Forty thousand, seven hundred ninety. Ninety thousand, four hundred six.

Seventy thousand, five hundred seven. Seventy thousand, ten. Thirty thousand, ninety-eight. Ninety thousand, one. Seventy thousand, three. Fifty thousand, five.

Four hundred thousand. Three hundred five thousand. Seven hundred thousand, five hundred.

Four hundred seven thousand, thirty. One hundred thousand, nine hundred eighty. Three hundred thousand, eight hundred seventy. Five hundred thousand, sixty.

Seven million, three hundred thousand. Four million, six hundred fifty thousand.

Eight million, thirty-seven thousand. Nine million, one thousand, eight hundred. Five million, five thousand, five.

Twenty-eight million. Seventy-three million. Nineteen million, two hundred thousand.

Eighty million, three hundred seventy thousand. Forty million, six hundred fifty thousand. Ninety million, eighteen thousand. Sixty million, four hundred five thousand, seventy. Fifty million, fifty thousand, fifty.

Six thousand, seventy. Thirty thousand, forty. Six hundred six thousand, six hundred. Eight hundred six thousand, forty.

2. Express in words the numbers:—

7000	8270	2804	7046	4080	5006	4506	2708
8100	2830	8205	6055	5020	3008	5056	3600
6400	7360	3706	5064	6070	1005	6007	4003
30000	82700	57360	80500	410000	208100	710560	
71000	27300	66450	20300	630000	703500	330070	
49000	73600	75540	70100	850000	307900	650080	
3200000	9018000	9008070	60040000	30100800			
5400000	2007300	6050040	20103000	60042001			
7600000	6004500	3020010	50709000	20034005			

Roman Notation.

1. Express by figures the following numbers:—

XIX	LXV	CX	CLX	DCXXIX.
XXI	LXIV	CXIX	CXC	DCCCXI.
X	LXXIX	C	CCLX	DCLX.
XLV	LXXXV	CXIV	CCXC	MCCLIX.

2. Express by letters the following numbers:—

50	56	100	200	400	500	700	800
60	65	120	230	440	560	785	909
70	78	109	209	409	614	704	937
80	87	148	348	547	626	875	990
90	99	167	367	579	649	869	1000

SECTION II.

ADDITION.

Problems.

Find the sum of the numbers to be added in each of the following problems; read each part, and each amount:—

(1)	(2)	(3)	(4)	(5)	(6)
9	234	89	7689	7	987
81	56	8675	68	89	65
645	7	986	9	7897	9
321	898	8	9876	654	5678
609	989	6879	989	5845	789

(7)	(8)	(9)	(10)	(11)
90830	78700	90509	38009	94949
68540	69800	30908	49008	85858
57650	56700	80407	68007	76767
86960	67800	40806	57006	67676
75670	78900	70305	76005	58585

(12)	(13)	(14)	(15)
70650 *acres.*	90250 *bales.*	30876 *tons.*	90876 *cords.*
7075 *acres.*	9087 *bales.*	9075 *tons.*	67 *cords.*
970 *acres.*	60543 *bales.*	87 *tons.*	6789 *cords.*
90875 *acres.*	98 *bales.*	56789 *tons.*	87654 *cords.*
25 *acres.*	456 *bales.*	860 *tons.*	567 *cords.*
9876 *acres.*	7089 *bales.*	6789 *tons.*	98765 *cords.*

Find the sum of—

16. 7495 + 789 + 65768 + 91827 + 89 + 9823.

17. 495867 + 7 + 78 + 728 + 79687 + 98596.

18. 800 feet + 8095 feet + 7084 feet + 80750 feet.

19. How many acres are 52000 acres, 75275 acres, 8725 acres, 875 acres, 8809870 acres, 89 acres, and 7025 acres?

20. 80750 cords + 825 cords + 70275 cords + 9875 cords + 42778 cords + 598 cords are how many cords?

21. How many dozen are 1800 dozen, 285 dozen, 47625 dozen, 84752 dozen, 9 dozen, 39 dozen?

22. 265 men + 7850 men + 10725 men + 1256 men + 7525 men + 875000 men are how many men?

23. Add together twenty thousand, three hundred four; eight hundred; nine thousand, two hundred; eight hundred ninety-seven; seven thousand, eight hundred nine; ninety thousand.

(24)	(25)	(26)	(27)	(28)
7865	198765 *cords.*	175250 *tons.*	987654	4938574
6789786	78968 *cords.*	985 *tons.*	394857	3847567
768987	7895 *cords.*	78695 *tons.*	837465	5867584
65764398	983746 *cords.*	8476 *tons.*	485766	9384756
59687	985 *cords.*	89 *tons.*	746587	3456789
75864738	98278 *cords.*	982736 *tons.*	596875	9876543
9287	8625 *cords.*	93847 *tons.*	693847	3948576
			738495	6978534
			926476	3849657

(29)	(30)		
728376 *bales.*	989 *feet.*	987654	7586943
84756 *bales.*	9786 *feet.*	394857	3849576
8976 *bales.*	79685 *feet.*	925689	6385749
98 *bales.*	928374 *feet.*	435465	9786534
9876 *bales.*	86429 *feet.*	786756	7984563
79685 *bales.*	9753 *feet.*	986787	3456987
827364 *bales.*	789 *feet.*	654345	6543456
		789876	7898765

(31)	(32)	(33)	(34)	(35)
709870	908077	200888	89	9283746
605430	605046	900876	7876	39485
405670	506075	700654	987654	89796756
908760	809084	400789	19283746	738495
506780	706053	500876	495867	6978
807650	607082	800789	8273	98729384
909870	806052	700996	64	75647

36. 785 sheep + 9755 sheep + 3875 sheep + 38755 sheep + 7865 sheep + 39485 sheep = how many sheep?

37. What is the amount of 725 dollars + 8250 dollars + 98 dollars + 75250 dollars + 126798 dollars + 596875 dollars + 785 dollars + 9753 dollars?

38. Add together 2070908 yards, 879456 yards, 34550 yards, 738275 yards, 29375 yards, 82756 yards, 378565 yards, 93765 yards, and 3579 yards.

39. How many bushels are 2075000 bushels + 750225 bushels + 5000000 bushels + 30728 bushels + 3948 bushels + 765 bushels?

40. How many dollars are 25750 dollars + 375225 dollars + 987753 dollars + 3746598 dollars + 9182736 dollars + 275 dollars?

41. How many are seven hundred million, seven hundred thousand, seven hundred; eighty million, eighty thousand, eighty; five million, five thousand, five; ninety-nine million, ninety-nine thousand, ninety-nine?

42. Benjamin Franklin was born in the year 1706, and lived to be 84 years old. When did he die?

43. In what year did a person born in the year 1814 become 57 years old?

44. By selling a house for 7225 dollars, a man lost 555 dollars. How much did the house cost?

45. A dealer bought a house for 8275 dollars, and sold it for 750 dollars more than he paid. What did he get for it?

46. How many feet will it take to go around a piece of ground which is 1789 feet long and 985 feet wide?

47. At a large foundry 9875 pounds of iron were cast each day for five days. How many pounds were cast in all?

48. A builder estimated the lumber to be used in a barn to be worth 1750 dollars, the carpenter-work 275 dollars, the hardware 116 dollars, and the painting 195 dollars. What was the whole calculated cost?

49. A flour-dealer shipped 3275 barrels of flour one month, 5129 the second, 2570 the third, and 1975 the fourth. How many barrels did he ship in all?

50. My farm cost me $7250; I built on it a house for $3575, a barn for $1786, and out-houses for $976. What was the entire cost?

51. I bought a store for $3775, paid $975 for enlarging it, $765 for repairs, and then sold it so as to gain $575. For how much did I sell it?

52. How many times does a clock strike which strikes the hours from noon till midnight?

53. Commencing with 767, find the sum of all the numbers to 776, inclusive. To 781. To 793. To 800.

54. What is the sum of all the numbers from 175 to 200, inclusive? From 985 to 1015?

55. Five ships arrived in port and landed 568 bales of cotton, 5257 bales, 3598 bales, 8765 bales, and 6675 bales. What was the whole number of bales?

56. A dry-goods dealer bought 1250 yards of silk for $1375, 890 yards of cloth for $2280, 785 yards of calico for $75, and 1500 yards of flannel for $137. How many yards did he buy, and what did he pay?

SECTION III.

SUBTRACTION.

Problems.

Find the difference between the numbers in the following problems; read each term and each remainder:—

(1)	(2)	(3)	(4)	(5)
758653	192830	648510	383740	536475
268564	183746	268717	283647	394857

(6)	(7)	(8)	(9)
307406 *tons.*	9012087 *cords.*	3405067 *acres.*	$807065
34785 *tons.*	706504 *cords.*	9868 *acres.*	6273

Find the remainder of—

10. 123045 — 7365; 516072 — 3456; 908375 — 8076.
11. 20903080 — 2030405; 30725809 — 10349072.
12. 342356457 — 52365598; 293847561 — 94758607.
13. 135056067 — 84545859; 701027341 — 90200732.
14. $230560 — $123559 are how many dollars?
15. How many feet are 350725 feet — 57275 feet?
16. Take 123783732 bushels from 143287890 bushels.
17. 307040654 quarts — 9073473 quarts = how many?
18. From 150723045 pounds take 98765432 pounds.
19. How many are 2098034 barrels — 909725 barrels?
20. Subtract 253702654 tons from 918374657 tons.
21. 340760560 yards — 234567468 yards = how many yards? 123040506 feet less 9080708 feet?

22. From three hundred million, five hundred six thousand, twenty-four take ninety million, fifty thousand, nine.

23. Subtract ninety million, eight hundred thousand, six hundred nine from seven hundred three million, fifty-five.

Find the difference and read the remainder of—

(24)	(25)	(26)	(27)	(28)	(29)
20908	40065	70000	100902	910080	730600
2079	9078	1009	90903	10092	40009

(30)	(31)	(32)
607005 *tons.*	5000705 *feet.*	308000 *men.*
90087 *tons.*	30908 *feet.*	9001 *men.*

33. 4000040 — 909; 50050050 — 80807; 600600 — 707.

34. 600040 — 605; 1000000 — 1001; 20000000 — 2002.

35. From 50050005 take 60606; and 505 from 40404.

36. How many are 3000000 less 30003? 4000000 less 5?

37. Subtract 25 from 25000000, and 750 from 150000000.

38. Find the difference between 605073 feet and 9645 feet.

39. 6336000 inches less 31680 inches are how many?

40. What is the remainder of $25000750 — $300875?

41. How many are 30050235 quarts — 60546 quarts?

42. 32525007 yards — 75000 yards are how many yards?

43. Take six million, sixty-six thousand, sixty-six from seventy million, fifty thousand, thirty.

44. Subtract four million, fifty thousand, fifty from seven hundred million, six hundred thousand, five hundred.

45. The minuend is eight hundred million, eight hundred thousand, eight hundred, and the subtrahend is seven million, seven thousand, seven. What is the difference?

46. How many tons are 200050000 tons less 725005 tons?

47. If the subtrahend is sixty million, six, and the minuend seven hundred million, seventy thousand, seventy, what is the remainder?

48. What is the remainder, if the minuend is three hundred thirty million, thirty thousand, thirty and the subtrahend is four million, forty thousand, forty-four?

49. What number increased by 23075 will become 10000000? What number increased by 50005?

50. What number must be taken from 7300705 to give a remainder of 97636?

51. What number must be added to 30075 to make 30075000? To make 500000005?

52. The sum of two numbers is 20050700, and the less number is 78205. What is the greater?

53. The sum of two numbers is 90050700, and the greater number is 70505075. What is the less?

54. America was discovered by Columbus in 1492. How many years have passed since that time?

55. The Revolutionary War commenced in 1775, and ended in 1783. How many years did it last?

56. Washington was born in 1732, became president in 1789, and died in 1799. How old was he when he became president, and how old when he died?

57. If you subtract 203040 ten times from 5000000, what will each remainder be?

58. One year a cotton-factory turned out goods that were worth $225000, and the expenses of making them were $192275. What were the profits?

59. A builder agreed to put up a row of houses for $125000. The materials and work cost him $116250. How much would he gain?

60. A farmer bought a farm for $12500, and paid on it $7525 cash. How much did he still owe?

61. Into a water-basin that could hold 5000000 gallons of water there ran 3010075 gallons. How nearly full was it?

62. The distance from the earth to the moon is about 240000 miles, and from the earth to the sun about 93000000 miles. How much further is it to the sun than to the moon?

63. A man put $13230 in two banks. If he put $9225 in one bank, how many dollars did he put in the other?

64. If a bushel of timothy-seed contains 41823360 grains, and a bushel of clover-seed 16400960, how many more grains of timothy-seed than of clover-seed are in a bushel?

65. If a city which contained 832507 inhabitants ten years ago now contains 935765, how many has it increased?

Problems Combining Addition and Subtraction.

NOTE.—Numbers connected by the sign + are to be united in one sum before subtracting the numbers connected or preceded by the sign —.

1. From the sum of 17395 and 4938 take 9378.

2. To the difference between 10724 and 7859 add 7294.

3. How much is the sum of 90807 and 909 diminished by 89017? Of 375919 + 909 — 99099?

4. What is the result of 10278 — 7289 added to 9827 + 10278? Of 90807 — 908 + 40608 — 709?

5. What number of feet of boards must be taken from 1725 feet to leave 500 feet more than 928 feet?

6. How many dollars must be added to $2750 to make a sum of money $275 less than $3265?

7. To the difference between 9753 yards and 3579 yards add their sum. From their sum take their difference.

8. The sum of three numbers is 8765 miles, and two of them are 2783 miles and 5296 miles. What is the third number?

9. If the sum of four numbers is $38765, and three of them are $12678, $9273, and $15987, what is the fourth?

10. A man was 21 years old in 1832, and he lived until he was 62 years old. In what year did he die?

11. A drover bought 750 sheep from one stock-farmer and 375 sheep from another, and then sold 467 sheep. How many had he remaining?

12. Three men bought a piece of ground for $8500. If the first paid $2725, and the second paid $2875, what remainder was there for the third to pay?

13. A merchant began business with $18450. In the first year he gained $1875, and in the second year he lost $3225. What was he then worth?

14. A cistern receives 1525 gallons by one pipe and 785 gallons by another, and discharges 1278 gallons by a third. How much does it then contain?

15. A real-estate agent bought a farm for $7500, put up fences that cost $275, and buildings that cost $1228, and sold it for $11524. How much did he gain?

16. An agent bought a piece of ground for $5000, put on it improvements that cost $675, and sold it at a loss of $487. What was the selling price?

17. A grain-dealer had 20015 bushels of grain after selling 8124 bushels and buying 9283 bushels. How many bushels had he at first?

18. A span of horses cost $1300, a carriage $450, and a set of harness $175. How much less than the horses did the carriage cost? The harness? The carriage and the harness? What was the entire cost?

19. If a man gave to his eldest son $7250, to the second son $775 less than to the eldest, and to the third $928 less than to the second, how much did each and all receive?

20. A flour-dealer bought 500 barrels of flour for $3125, and sold 217 barrels for $1725. How many barrels were left, and how much are they worth?

SECTION IV.

MULTIPLICATION.

Problems.

Find the product of—

1. 4075 × 135; 8064 × 531; 7503 × 357.
2. 19170 × 305; 28260 × 907; 37350 × 507.
3. 91827 × 2345; 82736 × 5432; 73645 × 3456.
4. What is the product of 8175 multiplied by 706?
5. How many are 192837 multiplied by 789?
6. 9761 × 456 = how many? 456 × 9167?
7. How many inches are 30000 inches × 345?
8. What is the product of 16500 feet × 1928?
9. The multiplicand is 91870, and the multiplier is 706. What is the product?
10. What is the product if the multiplier is 2376, and the multiplicand is 75 dollars?
11. Multiply thirty thousand, ninety-nine by four hundred fourteen; and by four thousand.
12. The multiplicand is seven thousand, ninety-eight feet, and the multiplier is three hundred five. What is the product?

13. Two factors are 405 and 7020. What is their product?

14. What is the product of the two factors 45 and 670?

15. If the earth moves around the sun at the rate of 1092 miles in a minute, how far does it move in 1440 minutes, or one day? In 7 days?

16. In a square mile are 640 acres. How many acres are in Pennsylvania, which contains 46000 square miles?

17. What would it cost to build 375 miles of a telegraph line, at $1725 per mile? At $2025 per mile?

18. A farm of 146 acres was sold at auction at $175 an acre. What was the selling price of the farm?

19. If a quantity of provisions will supply 960 soldiers 92 days, how many soldiers would it supply for one day?

20. Find the product of the three factors $23 \times 45 \times 67$.

21. If a ferry-boat carries on an average 68 passengers each trip, how many passengers does it carry in 6 days, making 15 trips a day? In 12 days? 18 days?

22. A farmer had 46 acres of wheat which yielded 28 bushels an acre. What was the crop worth at $2 a bushel?

23. How much money is needed to pay a regiment of 960 men for 12 months, if each man receives $16 a month?

24. What is the product of 23 times 45 times 678?

25. How many words are in a book of 328 pages, if it averages 32 lines on each page, and 12 words in each line?

26. If sound moves at the rate of 1142 feet in a second, how far will it move in 5 minutes of 60 seconds each?

27. If a horse requires 15 pounds of hay per day, how many pounds of hay are required to feed 6 horses during the 30 days of April? The 31 days of December?

28. A freight-train consists of 20 cars; each car contains

75 barrels of flour, and each barrel weighs 196 pounds. What weight of flour is in the cargo?

29. Multiply 34567 by 1234; by 2345; by 3456.

30. Multiply 5607089 by 5067; by 56078; by 70898.

31. Multiply 70809080 by 34560; by 46890; by 57930.

32. Multiply 24689000 by 357900; by 9876000.

33. What is the product if the multiplier is 987654, and the multiplicand is $24685900?

Problems in Addition, Subtraction and Multiplication.

NOTE.—Numbers connected by the sign × are to be multiplied first; then numbers or results connected by + are to be added; and, lastly, numbers or results preceded by — are to be subtracted.

1. What is the sum of 57 times 123 and 68 times 234?

2. What is the difference between 78 times 345 and 89 times 456?

3. What is the result of $642 \times 68 - 246 \times 86$?

4. Find the sum of $357 \times 57 + 579 \times 75$. The result of $6325 + 1875 - 125 \times 25$.

5. Multiply the sum of 305 and 970 by the difference between 1005 and 705.

6. What is the product obtained by multiplying the difference between $2050 and $905 by the sum of 725 and 280? The product of $2050 + $905 multiplied by 725 — 280?

7. Harry earned in a year 25 times 25 dollars less 275 dollars. What amount did he earn?

8. An engineer earns $60 a month, and his expenses are $625 a year. How much can he save in 2 years? How much in 15 years?

9. A pork-packer put up 800 barrels of pork that cost

him $12 a barrel, and sold them all at a gain of $1725. What was the selling price?

10. A man paid $2500 for the building of his house, and for his farm he paid 4 times as much, lacking $750. What was the cost of his property?

11. A cotton-merchant bought 436 bales of cotton at $125 a bale, and sold them at $144 a bale. How much did he gain?

12. If a drover who has 175 cattle should buy 87 more, and then sell them all at $45 each, how much would he receive for them?

13. The President of the United States receives a salary of $50000 a year. If his expenses are $100 a day, how much does he save in 365 days, or a year?

14. A company built 425 miles of railroad at $15257 a mile. After having paid $5000000, how much did the company still owe?

15. If a flour-dealer who buys 1500 barrels of flour at 9 dollars a barrel should pay cash $9000 and give his note for the balance, what would be the value of the note?

16. A banker has an income of $12750 a year; he pays $1500 for house-rent, and 5 times as much for family expenses. How much does he save annually?

17. A drover bought 125 cattle at $35 a head, and it cost him $6 a head to get them to market. If he sold them at $45 a head, how much did he gain or lose?

18. Two persons are 1200 miles apart, and walk towards each other, one going 35 miles a day, and the other 30 miles a day. How far apart will they be at the end of 15 days? At the end of 18 days?

19. A cleared in business one year $750, B 3 times as much, lacking $575, and C twice as much as A and B together. How much did B clear? How much did C clear? How much did all clear?

SECTION V.

DIVISION.

Problems.

Find the quotient of—

1. 4896 ÷ 4; 6785 ÷ 5; 8766 ÷ 6; 9961 ÷ 7.

2. 78984 ÷ 6; 89873 ÷ 7; 98760 ÷ 8; 10908 ÷ 9.

3. 9064 reams ÷ 8; 8298 quires ÷ 9; 9872 sheets ÷ 8.

4. 128979 bushels ÷ 9; 985768 quarts ÷ 8.

5. 106584 square yards ÷ 8; 679761 square feet ÷ 9.

6. Divide 255150 by 3, by 5, by 7, and by 9.

7. What is the quotient of 290439 feet ÷ 9?

8. The dividend is 292530, and the divisor is 7. What is the quotient?

9. What is the quotient, if the dividend is 256896 gallons, and the divisor is 8?

10. 3579 ÷ 7; 24680 ÷ 8; 97531 ÷ 8; 4680 ÷ 7.

11. 2947 cases ÷ 6; 38560 chests ÷ 7; 8642 sacks ÷ 8.

12. 37250 bales ÷ 7; 92720 pair ÷ 8; 46358 tons ÷ 9.

13. Divide 3151719 feet by 6, by 8, by 7, and by 9.

14. 7 times a certain number is 1020304. What is the number? 1020304 is 8 times what number?

What is

15. One-sixth of 237500?
16. One-eighth of 125725?
17. One-seventh of 37265?
18. One-ninth of 928650?
19. One-fourth of $12572?
20. One-sixth of 27852 rods?
21. One-fifth of 92837 feet?
22. One-ninth of 30502 tons?

23. If the dividend is 102750 feet, and the divisor is 8, what is the quotient? If the divisor is 9?

24. The product of two numbers is 2350275 tons, and the multiplier is 6. What is the multiplicand?

25. Find the quotient of seven million, twenty thousand, nineteen divided by seven; by six; and by nine.

26. Divide ten million, fifty thousand, four hundred by three; by five; by seven; and by nine.

Find one of the

27. 4 equal parts of 12505.
28. 6 equal parts of 23457.
29. 5 equal parts of 35790.
30. 7 equal parts of 46790.
31. 8 equal parts of 56789.
32. 9 equal parts of 13579.

33. The sum of the eight equal parts of a number is 287650 dollars. What is one of the parts?

34. If 8 miles of railroad cost $135000, what was the average cost per mile? If they cost $250000?

35. How many barrels of flour can be made from 2570250 bushels of wheat, if 5 bushels make one barrel?

36. A grain-dealer shipped 125000 bushels of wheat in 4-bushel sacks. How many sacks did he fill?

37. When sugar is worth $9 a hundred, how many hundred pounds can be bought for $135135?

38. The national debt of the United States was at one time $2300000000. How much would have been required each year to have paid it in 7 equal yearly payments?

LONG DIVISION.

Problems.

Find the quotient of—

1. 80706 ÷ 24; 50607 ÷ 26; 60504 ÷ 28; 40506 ÷ 30.
2. 97005 ÷ 33; 75003 ÷ 35; 53001 ÷ 37; 31009 ÷ 39.
3. 908060 ÷ 708; 807060 ÷ 809; 706050 ÷ 908.

4. 50000 yards ÷ 53; 70000 acres ÷ 57; $9000 ÷ 63.

5. 200304 pounds ÷ 234; 500790 barrels ÷ 567.

6. 305006 miles ÷ 456; 406335 cattle ÷ 789.

How many times is

7. 68 contained in 6985? In 60985? In 160985?

8. Find the quotient of 40500 divided by 89; by 809.

9. What remains after dividing $304005 by 876?

10. 4005006 square feet ÷ 890 gives what remainder?

11. Divide 608040 by 56; by 67; by 78; by 89.

12. Divide 6008004 by 506; by 607; by 708.

13. Find the quotient of thirty-two thousand, ninety-eight divided by nine hundred eighty-seven.

14. How many times does four million, four hundred thousand, contain one thousand, fourteen?

15. What is the quotient of thirty million, thirty thousand, thirty divided by two thousand, thirty-four?

16. How many remain after dividing sixty million, sixty thousand, six hundred by three thousand, fifty-six?

Find the quotient of—

17. 20405 ÷ 200; 90087 ÷ 400; 30560 ÷ 600.

18. 80076 ÷ 230; 45006 ÷ 340; 76050 ÷ 560.

19. 357000 pounds ÷ 2000; 1357090 tons ÷ 9000.

20. 246809 divided by 30, by 450, by 506, by 6700.

21. What is one-tenth of 375? One-hundredth of 7965?

22. What number is equal to 79500 ÷ 4000?

23. A carriage-maker received $15000 for light carriages at $200 each. How many carriages did he sell?

24. There are 480 sheets in a ream. How many reams will 259200 sheets make? How many will 242880 sheets make?

25. 640 acres make a square mile. How many square miles in Pennsylvania, which contains about 29440000 acres?

26. How many regiments averaging 750 men will make an army of 45000 men? An army of 64125 men?

27. At $1750 per car, how many cars can a railroad company buy with $52500? With $78750?

28. How many city lots at $2500 each can be bought for $227500? How many for $267500?

29. A cord of wood contains 128 solid feet. How many cords are in a pile containing 12750 solid feet?

30. The earth moves round the sun at the rate of about 11491200 miles in 168 hours. What is the rate per hour?

31. The average daily receipts of a ferry-boat are $275. In how many days will the receipts amount to $50000?

32. If 245 bushels of wheat weigh 14945 pounds, what is the average weight per bushel?

33. Mount Everest in Asia is about 29000 feet high. How many miles high is it, each mile being 5280 feet?

34. Divide 400070009 by 5607; by 6708; by 7809?

35. Divide 20030040 by 60708; by 70809; by 90705.

36. Divide 560780900 by 69584; by 72839; by 89076.

37. The product of two numbers is 99876540, and one of the numbers is 98765. What is the other?

38. The dividend is 8705003750, and the quotient is 96075. What is the divisor?

SECTION VI.

Miscellaneous Problems.

1. To 4 add 8, subtract 3, multiply by 5, divide by 9, subtract 5. What remains?

2. From 14 take 5, add 2, multiply by 6, divide by 3, subtract 7, multiply by 6. What is the result?

3. Divide 49 by 7, multiply by 6, subtract 7, add 15, divide by 10, add 8. Result?

4. The quotient is 10, the remainder is 8, and the divisor is 12. What is the dividend?

5. A grocer bought 7 barrels of flour at $6 a barrel. For how much a barrel must he sell it to gain $14 on the lot?

6. If you earn 9 dollars a week, and pay 5 dollars for board, and 2 dollars for other expenses, how much will you lay up in 8 weeks? How much in 12 weeks?

7. A trader bought 12 pairs of shoes at $3 and 6 hats at $4, and paid for them in 5-dollar bills. How many did it take? How many 10-dollar bills would it have taken?

8. If 5 oranges are worth 25 cents, and 6 lemons are worth 42 cents, how much more is a lemon worth than an orange. How much are an orange and a lemon worth?

9. A furniture-dealer bought 8 tables for $40, and sold them at $6 each. How much did he clear?

10. How many pounds of rice at 10 cents a pound will cost as much as 6 pounds of sugar at 15 cents?

11. If 5 masons lay a wall in 12 days, how many men would be required to lay it in 3 days? In 4 days?

12. What is the cost of 18 quarts of milk, at the rate of 40 cents for 5 quarts? The cost of 25 quarts?

13. A miller bought 10 bushels of wheat for $20, and I bought 12 bushels at the same rate. How much did I pay?

14. If 5 barrels of flour will pay for 10 bushels of wheat at $3 a bushel, what is the flour worth per barrel?

15. How many 4-quart cans of milk can be filled from five 8-quart cans? From six 12-quart cans?

16. In how many days can 7 men earn as much as 5 men can earn in 14 days? As much as 4 men in 21 days?

17. If four 10-dollar bills are paid for 8 yards of cloth, what is the price per yard?

18. A butcher bought 2 calves at $9 each, and one calf for $12. What was the average cost?

Find the value of—

19. $5 + 4 \times (9 - 6)$.*
20. $(5 + 4) \times 8 - 5$.
21. $4 + 3 \times (8 - 5)$.
22. $7 + (19 - 3) \times (4 - 3)$.
23. $7 + \overline{19 - 3} \div 4 - 3$.*
24. $7 + 19 - 3 \times 4 - 3$.

25. Find the sum of 65×76 and $6764 \div 89$.

26. From $7524 \div 76$ take $8765 - 8678$.

27. Multiply $7865 + 5678$ by $139725 \div 405$.

28. Find the value of $(75 + 25 \times 25 - 50) \times 205$.

29. What is the result of $174225 \div 345 + 20 \times 75$?

30. Divide the product of 98 times 765 by the sum of 1865 and 2035 less 3543.

31. Divide 13230 by 98, subtract the quotient from 76 times 67, and to the remainder add one-sixth of 258.

32. From 7531 take 1357, multiply the remainder by 25, and divide the product by 725.

33. A man bought 75 acres of land at $90 an acre, paid $325 for improvements, and then sold it for $8675. How much did he gain per acre?

34. A and B started on a journey, A traveling 29 miles

* Note.—When figures are enclosed in parenthesis marks, as $(9 - 6)$ Ex. 19, or marked with a vinculum or line over them, as $\overline{19 - 3}$, Ex. 23, the operation indicated is to be performed and the result used instead of the original figures thus, in Ex. 19 $(9 - 6) = 3$, and the example will then stand $5 + 4 \times 3$. Operations of multiplication and division must be performed before those of addition and subtraction, unless parenthesis indicate otherwise (see Notes pp. 14 and 18).

a day, and B 35 miles. In how many days will they be 84 miles apart. How soon will they be 96 miles apart?

35. If a mechanic earns $775 a year, and pays $240 for rent, and 390 for family expenses, how many years will it take him to save $2900?

36. If a mechanic earns $550 a year, and his expenses are $425, how many years will it take him to buy a farm of 50 acres at $75 an acre?

37. A dairy-man in packing 765 pounds of butter found that it filled 18 tubs, with 9 pounds left over. How much did he put in each tub?

38. If three steers weigh respectively 875 pounds, 944 pounds, and 1025 pounds, what is their average weight?

39. If a merchant's sales amount to $25225 in a week of 6 days, what are his average daily sales?

40. A farmer bought 25 acres at $80 an acre, and 30 acres at $75 an acre? What was the average price per acre?

41. How many casks, each holding 21 gallons, can be filled from 25 hogsheads of wine, containing 84 gallons each?

SECTION VII.

Properties of Numbers.

1. What is the product of the factors 3 and 3? 3 and 5?
2. The factors are 3 and 4. What is the product?
3. How many are 2 times 1? 3 times 1? 5 times 1?
4. How many ones in 3? In 5? In 7?
5. What numbers multiplied together produce 3? 5? 7?

6. What two numbers other than itself and 1 multiplied together produce 4? 6? 8? 10?

7. What two numbers other than itself and 1 divide 4 without a remainder? 6? 8? 10? 18? 24?

8. Of what number are 2 and 3 the factors? 2, 2, and 3? 2, 3, and 5? 2, 2, 3, and 5?

9. Name the smallest numbers other than 1 that will divide 6 without a remainder; 12; 30; 60.

10. Which are all the numbers that divide 9 without a remainder? 12? 18? 20? 24?

11. Name five numbers each of which is produced only by multiplying together itself and 1.

12. Name five numbers each of which is produced by multiplying together other numbers than itself and 1.

13. Name the numbers from 0 to 20 that can be divided by 2 without a remainder.

14. Name all the numbers from 0 to 20 that cannot be divided by 2 without a remainder.

Definitions.

163. The ***Factors*** of a number are those numbers which, multiplied together, produce that number.*

Thus, *2* and *3* are *factors* of *6*; for, being multiplied together they produce 6; so, also, 2, 3, 5 are the factors of 30.

Note. — The terms *number*, *factor*, *divisor*, and *multiple* in this Section are used in the sense of *integers*.

164. A ***Prime Number*** is a number that has no other factor than 1 and itself.

Thus, *5* is a *prime number*; so, also, are 7, 11, 13, 17, etc.

165. A ***Composite Number*** is a number that has other factors than 1 and itself.

* Sections numbered 1–162, inclusive, will be found in Part I. of the Model Elementary Arithmetic.

Thus, *15* is a *composite number*, since $15 = 3 \times 5$; and 20, since $20 = 5 \times 4$, or 10×2, etc.

166. An ***Even Number*** is a number that can be divided by 2 without a remainder.

All even numbers end with 0, 2, 4, 6, or 8.

167. An ***Odd Number*** is a number that cannot be divided by 2 without a remainder.

All odd numbers end with 1, 3, 5, 7, or 9.

Factors, or Divisors.

1. Name all the prime numbers from 0 to 20.

2. Name all the composite numbers from 0 to 20.

3. What is the product of the prime numbers 2 and 2? 2 and 3? 3 and 5? 2, 2, and 3? 2, 3, and 5?

4. Of what prime numbers is 4 the product? 6? 15? 12? 30? 16? 33? 18? 36?

5. What prime numbers are factors of 18? 24? 36?

6. Name the smallest exact divisors of 27; 32; and 42.

7. Name all the smallest prime numbers whose continued product is 36.

8. Name all the smallest prime numbers that will exactly divide 36.

9. Name all the numbers which, multiplied together, produce 12; 36; 48.

10. Name all the numbers that will exactly divide 12; 36; 48.

11. Name two composite numbers that are factors of 48. Name the smallest factors of these two composite numbers.

12. Name all the prime factors of 48. Name every factor of 48.

Definitions.

168. An ***Exact Divisor*** is a number that divides any given number without a remainder.

Thus, 5 is an exact divisor of 25; and 8 of 56.

1. The exact divisors of a number are also the factors of that number.

2. The terms *Factor* and *Divisor* differ only in use; *factor* suggesting the process of multiplication, and *divisor* the process of division.

Note. — A number exactly divides another when it is contained in that other without a remainder.

Thus, 5 exactly divides 10, 15, 30, 75, etc.

A number can be exactly divided by another when it contains that other without a remainder.

Thus, 24 can be exactly divided by 2, 3, 4, 6, 8, and 12.

169. A number can be exactly divided by 2 when it ends with 0, 2, 4, 6, or 8.

Thus, 10, 22, 34, 456, 678 can be exactly divided by 2 (166).

170. A number can be exactly divided by 5 when it ends with 5 or 0.

Thus, 15, 70, 345, 670 can be exactly divided by 5.

171. A number can be exactly divided by 3 when the sum of the units expressed by its figures can be divided by 3.

Thus, 3579 can be exactly divided by 3, since the sum of 3, 5, 7, 9, or $3 + 5 + 7 + 9 = 24$, can be divided by 3.

172. A ***Prime Factor*** is a factor that is a prime number (164).

Thus, 5 is a prime factor of 15; and 7 of 42.

173. A ***Composite Factor*** is a factor that is a composite number (165).

Thus, 6 is a composite factor of 12; and 14 of 42.

174. ***Factoring*** is the process of separating a composite number into its prime factors.

Since 1 is a factor of every number, it is not regarded either in naming or in finding the prime factors of numbers.

175. Principles.

I. *Every composite number is the product of all its prime factors.*

II. *Every factor of a number is an exact divisor of that number.*

III. *Every number can be exactly divided only by its prime factors, and by the product of any two or more of them.*

Written Exercises.

176. *Example* 1. — What are the prime factors, or divisors, of 456?

SOLUTION.

$2)456$
$2)228$
$2)114$
$3)57$
19

EXPLANATION. — Since the given number is an even number, it can be divided by the prime factor 2 (169).

For the same reason, divide the quotient, 228, by 2; and the next quotient, 114, by 2.

Since the sum of 5 and 7 can be exactly divided by 3, divide 57 by 3 (171), giving the quotient 19, which is a prime number, and cannot be separated into factors (164).

PROOF.

$2 \times 2 \times 2 \times 3 \times 19 = 456$

Hence, the divisors 2, 2, 2, 3, and 19 are all the prime factors of 456.

177. *Example* 2.—Find all the factors, or divisors, of 90.

SOLUTION.

$2)90$
$3)45$
$3)15$
5

$2, 3, 3, 5$
$2 \times 3 = 6$
$2 \times 5 = 10$
$3 \times 3 = 9$
$3 \times 5 = 15$
$2 \times 3 \times 3 = 18$
$3 \times 3 \times 5 = 45$

EXPLANATION. — Since the given number is an even number, etc.

Hence, 2, 3, 3, and 5 are the prime factors of 90.

Since the only composite factors of 90 are the products of any two or more of its prime factors, 6, or 2 times 3; 10, or 2 times 5; 9, or 3 times 3; 15, or 3 times 5; 18, or 2 times 3 times 3; and 45, or 3 times 3 times 5, are the composite factors of 90.

Hence, 2, 3, 3, 5, 6, 9, 10, 15, 18, and 45 are all the factors, or divisors, of 90 (175, III.).

178. Rule to find Prime Factors, or Divisors.

I. *Divide the given number by any of its prime factors greater than* 1.

II. *Divide the quotient by any other of its prime factors; and so proceed until a quotient is obtained that is a prime number. The several divisors and the last quotient will be the prime factors required.*

Problems.

Separate into their prime factors —

1. 12, 14, 16, 18, 20, 24.
2. 15, 21, 27, 33, 39, 45.
3. 28, 32, 40, 50, 54, 63.
4. 63, 75, 81, 84, 88, 96.
5. 108, 112, 132, 135, 140.
6. 144, 160, 176, 192, 196.
7. 210, 215, 275, 360, 375.
8. 450, 525, 630, 760, 840.

9. Resolve into prime factors 1265, 1925, 286, and 3094.

10. What prime numbers are divisors of 3927 and 6380?

11. What prime numbers exactly divide 7264 and 8575?

12. Find the prime divisors of 10010, 11781, and 12673.

13. What are the prime factors of 25984, 36504, 51975?

14. Find the prime factors of 15015, 24255, and 40425.

15. Of what prime factors is 45045 the product?

16. What prime numbers multiplied together produce 19404? What produce 389844?

17. Find every prime and composite factor of 24 and 54.

18. Find all the different values of articles that can be bought for 96 cents.

19. Find which numbers between 50 and 100 are prime and which are composite.

20. Which of the numbers 359, 441, 577, 651, and 1152 are prime and which are composite?

Common Factors, or Divisors.

1. What is the product of the factors 3 and 5? 3 and 7?

2. What are the factors of 15? Of 21? Of 33?

3. What factor is common to 15, 21, and 33?

4. What numbers divide 15 without a remainder? 21? 33?

5. What number exactly divides the three numbers 15, 21, and 33?

6. What factor, or divisor, then, is common to 15, 21, and 33? What factor is common to 14, 35, and 49?

7. Name two numbers of which 5 is a common divisor. Three, of which 5 and 7 are common divisors.

8. Name an exact divisor common to 20 and 30.

9. What are the prime factors of 16? Of 24? What are the composite factors?

10. What prime factors have 18 and 24 in common? Tell the products of these common prime factors.

11. What is the greatest factor that will exactly divide 18 and 27? 24 and 36? 32, 40, and 48?

12. What is the greatest divisor common to 16 and 24? 18 and 27? 24 and 36? 32, 40, 48?

13. What exact divisor is common to 7 and 3 times 7? To 11 and 5 times 11?

14. What divisor is common to 6 and 9? To 6 and 9, and 54, their product?

15. What divisor is common to 18 and 24, and 42, their sum? To 18 and 24, and 6, their difference?

Definitions.

179. A ***Divisor,*** or ***Measure,*** is a number that divides any given number without a remainder.

Thus, *6* is a *divisor*, or *measure*, of *18*, since 6 divides 18 without a remainder.

180. A ***Common Divisor*** is a number that divides each of two or more given numbers without a remainder.

Thus, *3* is a *common divisor* of *18* and *24*, since it divides each of them without a remainder.

A factor of each of two or more numbers is a *common factor* of those numbers.

181. The ***Greatest Common Divisor*** is the greatest number that divides each of two or more given numbers without a remainder.

Thus, *6* is the *greatest common divisor* of *18* and *24*, since it is the greatest number that divides each of them without a remainder.

182. Numbers *are prime to each other* when they have no common factors, or divisors.

Thus, 7 and 12 are prime to each other; so, also, are 9 and 16; 16 and 25; 9, 16, and 25.

183. Principles of Common Divisors.

I. *A factor common to two or more numbers is a common divisor of those numbers.*

II. *The only common divisors of two or more numbers are their common prime factors, and the products of any two or more of them.*

III. *The greatest common divisor of two or more numbers is the product of all their common prime factors.*

IV. *The common divisor of two numbers is also a divisor of their sum and of their difference.*

Written Exercises.

184. *Example* 1. — Find all the common divisors of 24 and 36.

SOLUTION.

$24 = 2 \times 2 \times 2 \times 3$

$36 = 2 \times 2 \times 3 \times 3$

Prime div. $= 2, 2, 3.$

$2 \times 2 = 4$

$2 \times 3 = 6$

$2 \times 2 \times 3 = 12$

Composite com. div. $= 4, 6, 12.$

EXPLANATION. — By separating 24 and 36 into their prime factors, 2, 2, and 3 are found to be common factors of 24 and 36, and are, therefore, common divisors of 24 and 36 (Prin. I).

Since the several products of any two or more prime factors

of a number are divisors of that number (Prin. II.), the several products that can be formed from the common factors 2, 2, and 3, must be all the common divisors of 24 and 36. Now 4, or 2 times 2; 6, or 2 times 3; and 12, or 2 times 2 times 3, are the several products of the common prime factors. Hence, 2, 2, 3, 4, 6, and 12 are all the common divisors of 24 and 36.

185. *Example* 2. — What is the greatest common divisor of 36, 63, and 81?

FIRST SOLUTION.

$36 = 2 \times 2 \times 3 \times 3$
$63 = 3 \times 3 \times 7$
$81 = 3 \times 3 \times 3 \times 3$
Com. div. $= 3$ *and* 3
Greatest com. div. $3 \times 3 = 9$

EXPLANATION. — By separating 36, 63, and 81 into their prime factors, 3 and 3 are found to be the only prime factors common to these numbers (Prin. I.).

Since the product of all the common prime factors of two or more numbers is their greatest common divisor, 9 or 3×3 is the greatest common divisor of 36, 63, and 81 (Prin. III.).

Second Method.

186. *Example.* — Find the greatest common divisor of 36, 63, and 81.

SECOND SOLUTION.

36)63(1
36
—
27)36(1
27
—
9)27(3
27
—

9)81(9
81
—

$9 =$ ***G. c. d.*** *of 36, 63, and 81.*

EXPLANATION.—First use two of the given numbers.

The greatest common divisor of two or more numbers cannot be greater than the least of the numbers (181).

Since any number is the greatest divisor of itself, if 36 is a divisor of 63, it is the greatest common divisor of 36 and 63. But 36 does not exactly divide 63, since there is a remainder of 27. 36 is not, therefore, a common divisor.

The common divisor of 36 and 63 is also a divisor of their difference, 27 (Prin. IV.); and if 27 is a divisor of 36, it is also a divisor of 63, which equals $(36 \times 1) + 27$. But 27 is not a divisor of 36, since there is a remainder of 9.

The common divisor of 27 and 36 is also a divisor of their difference, 9 (Prin. IV.); and if 9 is a divisor of 27, it is also a divisor of 36, which equals $(27 \times 1) + 9$. 9 exactly divides 27. 9 is, therefore, a common divisor of 36 and 63, since it divides 27; and, therefore, 36, which equals $(27 \times 1) + 9$; and if it divides 36, it also divides 63, which equals $(36 \times 1) + 27$.

9 is also a divisor of 81, and is, therefore, the greatest common divisor of 36, 63, and 81.

187. Rule to find the Greatest Common Divisor of two or more Numbers.

When two numbers are given,

I. *Divide the greater number by the less; the first divisor by the first remainder; the second divisor by the second remainder; and so on until there is no remainder.*

II. *The last divisor will be the greatest common divisor of the two numbers.*

When more than two numbers are given,

I. *Find first the greatest common divisor of two of the numbers; then the greatest common divisor of this divisor and one of the other numbers; and so on until all the numbers are used.*

II. *The last divisor will be the greatest common divisor of all the numbers.*

Problems.

Find the common divisors of—

1. 6 and 9; 10 and 15.
2. 9 and 12; 6 and 21.
3. 14 and 21; 15 and 24.
4. 18 and 27; 20 and 25.
5. 21 and 35; 25 and 40.
6. 28 and 49; 39 and 52.
7. Find the common factors of 20, 25, and 30.
8. Find the prime factors common to 27, 36, and 45.
9. What number of feet will exactly divide 39 feet, 52 feet, and 65 feet?

Find the common and the greatest common divisors of

10. 12 and 18; 20 and 30.
11. 28 and 42; 36 and 48.
12. 32 and 64; 56 and 72.
13. 66 and 80; 75 and 90.
14. 100 and 120; 126 and 200.
15. 105 and 135; 169 and 240.

16. How many common divisors have 60, 75, and 90?

17. Find every common divisor of 105, 140, 175, and 210.

18. Find every number of miles that will exactly divide 210 miles, 270 miles, and 390 miles.

19. Find the size of every box that can be filled an exact number of times by 315 bushels of wheat, and 630 bushels

Find by the second method the greatest common divisor of—

20. 35 and 49.
21. 42 and 56.
22. 56 and 80.
23. 63 and 91.
24. 81 and 117.
25. 96 and 145.
26. 105 and 195.
27. 144 and 276.
28. 279 and 378.
29. 336 and 484.
30. 476 and 578.
31. 580 and 696.

32. 651 and 775; 777 and 925; 816 and 924; 912 and 960.

33. 1050 and 1215; 2058 and 2163; 3640 and 4550.

34. 5148 and 6435; 678 and 7072; 7980 and 8113.

35. 5355 and 10239; 858 and 17732; 11664 and 16038.

36. 48, 72, and 84; 42, 70, and 91; 64, 80, and 98.

37. 132, 148, and 183; 216, 252, and 297.

38. 399, 484, and 589; 504, 672, and 1029.

39. 1056, 1368, and 1766; 2268, 3402, and 4509.

40. 5280, 6666, and 10791; 144, 198, 228, and 297.

41. 312, 546, 676, and 819; 1008, 2205, 4452, and 17829.

42. $376, $693, and $858; 650 tons, 1625 tons, and 3275 tons; 858 feet, 1287 feet, and 1484 feet.

43. What is the greatest divisor common to 1504 miles, 3196 miles, 4606 miles?

44. What is the greatest number of yards that can be a common factor of 6396 yards, 8502 yards, and 14937 yards?

45. Find the greatest number of feet that will exactly divide 5280 feet, 7020 feet, and 9715 feet.

46. What is the greatest length of rails that can, without cutting, be used in fencing the road front of three lots measuring 209 feet, 363 feet, and 187 feet?

47. In a house are three rooms: the first is 16 feet wide, the second 20 feet, and the third 24 feet. What is the greatest width of matting that will exactly fit each?

48. Frank has 35 blocks, Thomas has 50, and Charles 65. How can they arrange them in groups so that each boy shall have the same number in each group?

49. What is the greatest number of melons that you could buy with 90 cents, 135 cents, and 210 cents, so as to get the same number of each kind?

50. A farmer put 66 bushels of corn, 90 of wheat, and 120 of oats into the largest possible sacks of equal size. What did each sack contain?

Multiples, or Dividends.

1. What is the product of the factors 3 and 5? 3 and 7? 5 and 7?

2. Name the products between 3 and 35 of which 3 is a factor. Of which 5 is a factor. Of which 7 is a factor.

3. Name the numbers between 3 and 35 that can be exactly divided by 3; by 5; by 7.

4. What are the prime factors of 6? Of 2 times 6? Of 4 times 6? Of 5 times 6?

5. Name all the prime factors of 4. Of 6. Of 4 times 6. Of 8 and 4 times 8. Of 9 and 6 times 9.

6. What prime factors are common to 4 and 6? To 6 and 8? To 6 and 9?

7. 10 is an exact number of times what two numbers? By what two numbers can 10 be exactly divided?

8. Name three numbers of which 2 and 3 are common factors; 3 and 5; 5 and 7.

9. Name three numbers each of which is an exact number of times 2 and 3; 3 and 5; 5 and 7.

10. Name three numbers each of which can be exactly divided by 2 and 3; 3 and 5; 5 and 7.

11. What is the least number of which 3 and 4 are factors? 3 and 5? 3, 4, and 5?

12. What is the least number that can be divided by 4 and 6? 6 and 8? 4, 6, and 8?

Definitions.

188. A ***Multiple,*** or ***Dividend,*** is a number that can be divided by a given number without a remainder.

Thus, *12* is a *multiple* of *4*, since it can be divided by 4 without a remainder. 24 is a multiple of 6.

189. A ***Common Multiple*** is a number that can be divided by each of two or more numbers without a remainder.

Thus, *12* is a *common multiple* of *3* and *4*, since it can be divided by 3 and by 4 without a remainder. So, also, is 24.

190. The ***Least Common Multiple*** is the least number that can be divided by each of two or more numbers without a remainder.

Thus, *12* is the *least common multiple* of *3* and *4*, since it is the least number that can be divided by 3 and by 4 without a remainder.

Note.—The terms *Multiple*, or *Product*, and *Dividend* differ only in use; *multiple* suggesting the process of multiplication, and *dividend* the process of division.

191. Principles of Common Multiples.

I. *A multiple of a number contains all the prime factors of that number.*

II. *A common multiple of two or more numbers contains all the prime factors of each of those numbers.*

III. *The least common multiple of two or more numbers is the least number that contains all the prime factors of each of those numbers.*

Written Exercises.

192. *Example* 1. — Find a common multiple of 6 and 8.

SOLUTION.

$6 \times 8 = 48$

$48 =$ ***Com. mult.***

EXPLANATION. — Since a common multiple of 6 and 8 is a number of which 6 and 8 are factors, or divisors, their product, 48, is the common multiple of 6 and 8; as is also any exact number of times 48 (188).

193. *Example* 2. — Find the least common multiple of 6, 8, and 12.

SOLUTION.

$6 = 2 \times 3$

$8 = 2 \times 2 \times 2$

$12 = 2 \times 2 \times 3$

$2 \times 2 \times 3 \times 2 = 24$

$24 =$ ***L. c. m.***

EXPLANATION. — First separate the given numbers into their prime factors. Now the least common multiple cannot be less than 12, the greatest of the given numbers, since it must contain 12, and, therefore, all the prime factors of 12, which are 2, 2, and 3 (Prin. I.).

The least common multiple of 12, 6, and 8 must contain also the prime factors of each of the numbers, 6 and 8, as well as of 12.

Of the factors of 8, 2 and 2 are common also to 12; they are, therefore, omitted as factors of the least common multiple, and only 2, the remaining factor, is retained.

Of the factors of 6, 2 and 3 are found also in the other two numbers, 8 and 12, and are, therefore, not used a second time.

Hence, 2, 2, 3, and 2 are all the prime factors of the given numbers, and their product, 24, is the least common multiple required (Prin. III.).

191. Principles of

I. *A multiple of a number* [illegible] *that number.*

II. *A common multiple of* [illegible] *the prime factors of* [illegible]

III. *The least common multiple of* [illegible] *the least number that contains* [illegible] *those numbers.*

Written

192. Example 1.—[illegible]

SOLUTION. [illegible]

$6 \times 4 = 24$ [illegible]

$48 = 6$ com. mult. [illegible]

exact number of times [illegible]

193. Example 2.—[illegible] 6, 8, and 12.

SOLUTION. [illegible]

$6 = 2 \times 3$

$8 = 2 \times 2 \times 2$

$12 = 2 \times 2 \times 3$

$2 \times 2 \times 3 \times 2 = 24$

$24 = $ L. c. m.

The least common [illegible] prime factors of [illegible]

Of the factors [illegible] times, united a [illegible] remaining factor [illegible]

Of the factors [illegible] twice, 9 and 2[illegible]

Exam[illegible]

2.

50.

100.

, 33.

68 rods.

tons; 18

08 pounds.

cords; 171

nd 100 acres.

5 and 10395

ultiple of 16

nd 3125 feet

Second Method.

194. *Example.* — Find the least common multiple of 30, 45, and 75.

SOLUTION.

$3)30,\ 45,\ 75$

$5)10,\ 15,\ 25$

$2,\ 3,\ 5$

$3 \times 5 \times 2 \times 3 \times 5 = 450$

$450 = L.\ c.\ m.$

EXPLANATION.—Write the numbers in a horizontal line.

Since 3 is a prime factor common to two or more of the given numbers, it is a factor of the least common multiple (Prin. III.).

Divide by the factor 3, and write the remaining factors, or the quotients, in a line beneath.

Since 5 is a prime factor common to two or more of the numbers, divide by 5, and write the quotients in a line beneath.

Since 2, 3, and 5 have no common factor, they are prime to each other. Therefore, the divisors, or factors, 3 and 5, and the last quotients, 2, 3, 5, are all the prime factors of the given numbers, and their product, 450, is the least common multiple required (Prin. III.).

195. Rule to find the Least Common Multiple of two or more Numbers.

I. *Place the given numbers in a horizontal line.*

II. *Divide by any prime number that is a factor of two or more of them, and write the quotients and the undivided numbers in a line beneath.*

III. *Divide, if possible, by any other prime factor common to two or more of them; and so continue to divide until the quotients and the undivided numbers are prime to each other.*

IV. *Multiply together the divisors, and the quotients and undivided numbers in the last horizontal line, and their continued product will be the least common multiple required.*

Note 1. — The multiple of a number is the number itself, or any number of times the number.

Note 2. — The common multiple of two or more numbers is their product, or any number of times that product.

Problems.

Find a common multiple of—

1. 2, 3, and 4.	4. 5, 7, and 11.	7. 9, 11, and 14.
2. 3, 5, and 7.	5. 6, 7, and 13.	8. 8, 12, and 15.
3. 4, 7, and 9.	6. 7, 9, and 10.	9. 11, 12, and 13.

10. \$4, \$8, \$10; 5 yards, 6 yards; 7 years and 8 years.

11. What number of miles is a common multiple of 6 miles, 8 miles, 9 miles, 10 miles?

12. Find a number of acres that is a common multiple of 7 acres, 8 acres, 9 acres, and 12 acres.

13. Find a number of tons that can be exactly divided by 8 tons, 10 tons, 14 tons, and 15 tons.

Find by the second method the least common multiple of—

14. 2, 4, and 6.	18. 8, 10, and 15.	22. 8, 32, and 48.
15. 3, 6, and 9.	19. 9, 12, and 18.	23. 9, 27, and 63.
16. 4, 6, and 8.	20. 6, 15, and 20.	24. 12, 42, and 72.
17. 6, 8, and 9.	21. 7, 21, and 28.	25. 15, 35, and 60.

26. 24, 8, 36, and 72; 35, 14, 21, and 105.

27. 13, 78, 52, and 104; 12, 4, 24, 8, 36, and 96.

28. 84, 7, 21, 63, and 6; 35, 7, 105, 5, 140, and 70.

29. 8, 2400, 160, 96, and 72; 2100, 105, 7, 21, and 100.

30. 19, 133, 38, 57, 95, 76; 121, 22, 132, 12, 1100, 33.

31. \$10, \$50, \$25, and \$100; 17 rods, 100 rods, and 68 rods.

32. 13 tons, 91 tons, 65 tons, 130 tons, and 195 tons; 18 pounds, 27 pounds, 144 pounds, 81 pounds, and 108 pounds.

33. 117 cords, 351 cords, 39 cords, and 1053 cords; 171 acres, 185 acres, 228 acres, 19 acres, 1900 acres, and 100 acres.

34. Find the least common multiple of 8085 and 10395; of 2310 and 27720; of 1152 and 9216.

35. What number is the least common multiple of 162 acres, 1194 acres, and 597 acres? Of 1875 feet and 3125 feet?

36. What is the least number that can be divided by each of the nine digits without a remainder?

37. What is the shortest piece of wire that can be divided into lengths of 5 feet, 125 feet, 15 feet, or 75 feet?

38. What is the least number of marbles that can be equally divided among 18, 30, or 36 boys?

39. What is the least number of acres in a farm that can be divided into fields containing 12, 16, 18, or 20 acres each?

40. Find the least number of quarts of milk that can exactly fill 8-quart, 12-quart, 15-quart, or 24-quart cans.

41. What is the smallest sum of money that would exactly purchase mules at $75 a head, cows at $50, or horses at $125?

42. What is the least number of cents with which you could buy an exact number of lemons at 6 cents each, oranges at 8 cents, bananas at 10 cents, or melons at 16 cents?

Cancellation.

1. What are the factors of 6? Of 18?

2. What factor remains if 2, one of the factors of 6, is omitted? If 3, one of the factors of 18, is omitted?

3. What effect upon the value of a number is produced by omitting one of the factors of that number?

4. What is the quotient of 18 divided by 6? Of 18 divided by 6 ÷ 2? Of 18 divided by 6 ÷ 3?

5. How many times greater is the quotient of 18 divided by 6 ÷ 3, than the quotient of 18 divided by 6?

6. What is the effect upon the quotient when a factor is omitted from the divisor?

7. What is the quotient of 24 divided by 4? Of 24 ÷ 2 divided by 4?

8. How many times less is the quotient of $24 \div 2$ divided by 4, than the quotient of 24 divided by 4?

9. What is the effect upon the quotient if a factor is omitted from the dividend?

10. What is the quotient of $18 \div 3$ divided by $6 \div 3$? Of $24 \div 2$ divided by $4 \div 2$?

11. What is the effect upon the quotient if the same factor is omitted from both divisor and dividend?

12. Name the factors of 12; of 18; the factors common to 12 and 18.

13. If the factors common to 12 and 18 are omitted, what is the quotient of the remaining factors?

14. If the common factors of 2 times 3 times 5 and 2 times 3 are omitted, what is the quotient of the remaining factors?

15. What is the quotient of $8 \times 7 \times 5$ divided by $4 \times 7 \times 5$?

Definition.

196. Cancellation is the process of shortening operations by striking out common factors from both divisor and dividend, and using only the remaining factors.

197. Principles of Cancellation.

I. *Striking out a factor from a number divides the number by that factor.*

II. *Striking out a common factor from the divisor and the dividend does not change the quotient.*

Written Problems.

198. *Example* 1. — Divide 147 by 21.

SOLUTION.

$$\frac{147}{21} = \frac{\cancel{7} \times \cancel{3} \times 7}{\cancel{7} \times \cancel{3}} = \frac{7}{1} = 7$$

EXPLANATION.—As in division, write the dividend above, and the divisor below the line.

Separate the dividend into its prime factors, 7, 3, and 7; and the divisor into its prime factors, 7 and 3. By striking out, or cancelling, the common factor 7, and the common factor 3 (Prin. II.), there remain in the dividend the factor 7, and in the divisor 1. Hence, the quotient is 7.

199. *Example* 2. — Divide $9 \times 75 \times 39$ by 13×54.

SOLUTION.

$$\frac{\overset{3}{\cancel{9}} \times 75 \times \overset{1}{\cancel{39}}}{\underset{2}{\cancel{13}} \times \underset{6}{\cancel{54}}} = \frac{75}{2} = 37\tfrac{1}{2}$$

EXPLANATION. — Write the dividend above and the divisor below the line. First cancel the common factor 9 from the dividend and from 54 in the divisor, leaving the factor 6 in the divisor.

Next cancel the common factor 13 from 13 and 39, leaving 3 in the dividend.

Next cancel 3 from 3 and 6, leaving 2 in the divisor.

The uncancelled factor in the dividend is 75, and in the divisor is 2, and $75 \div 2 = 37\frac{1}{2}$, the quotient required (Prin. II.).

200. Rule for Cancellation.

I. *Write the numbers forming the dividend above a horizontal line, and the numbers forming the divisor below the line.*

II. *Cancel all factors common to the divisor and the dividend.*

III. *Divide the product of the uncancelled factors of the dividend by the product of the uncancelled factors of the divisor. The quotient will be the result required.*

Problems.

Divide by cancellation —

1. 255 by 60; 819 by 441; 3744 by 2952.
2. 3765 by 2625; 7056 by 6068; 24375 by 5625.
3. $15 \times 6 \times 7$ by 11×8; 36×7 by 27×11.
4. $3 \times 26 \times 14$ by 84; $4 \times 3 \times 9$ by $10 \times 4 \times 15$.
5. $16 \times 5 \times 4$ by 20×8; $21 \times 11 \times 26$ by 14×13.

6. $7 \times 9 \times 15$ by 9×5; $4 \times 5 \times 56$ by $8 \times 7 \times 5$.

7. $36 \times 11 \times 7$ by $5 \times 14 \times 9$; $56 \times 7 \times 12 \times 3$ by $6 \times 14 \times 8$; $51 \times 11 \times 4$ by $17 \times 8 \times 44$.

8. $75 \times 2 \times 10$ by $2 \times 11 \times 5$; $81 \times 7 \times 8$ by 27×28.

9. $7 \times 9 \times 45$ by $7 \times 9 \times 9$; $5 \times 5 \times 125$ by 25×25.

10. Divide 28×32 by $7 \times 8 \times 9 \times 16$; 560×100 by 80×25; $26 \times 56 \times 14$ by 910×2.

11. What is the value of $(306 \times 45 \times 61) \div 200 \times 60 \times 91$? Of $50 \times 19 \times 8 \div (150 \times 2 \times 95)$?

12. How many times is 27×35 contained in 90×7?

13. Find the quotient of $35 \times 51 \times 7 \div 7 \times 102 \times 14$.

14. The dividend is $3 \times 9 \times 15 \times 21$, and the divisor is $15 \times 3 \times 5 \times 7$. What is the quotient?

15. The factors of a dividend are 5, 35, and 49, and of a divisor are 25, 7, and 14. What is the quotient?

16. How many pounds of tea at 75 cents a pound must be given in exchange for 5 pounds of coffee at 30 cents?

17. How many barrels of flour at 9 dollars a barrel are worth as much as 18 tons of coal at 7 dollars a ton?

18. How much corn at 90 cents a bushel will pay for 120 yards of muslin at 15 cents a yard?

19. If 7 acres of land are worth \$350, what will 150 acres cost?

20. At \$21 a suit, how many suits can be made out of 4 pieces of cloth, each containing 24 yards worth \$3 a yard?

Review Problems.

1. Find the prime numbers between 100 and 150. The composite numbers between 150 and 200.

2. Find the prime factors of 16580, 19475, and 24992.

3. What are all the prime divisors of 45744 and 62872?

4. Find the prime factors common to 216, 504, 726.

5. Find all the different divisors of 288 feet and $564.

6. Find every divisor common to 288 and 384.

7. Which of the numbers 739, 882, 997, and 8085 are prime? Which are composite, and what are their factors?

8. What is the greatest common divisor of 27924 and 41276? Of 31059, 44631, and 59508?

9. Find the least common multiple of 7, 168, 28, 70, 112, and 224. Of 249, 1245, 1743, and 3984.

10. Find every prime factor of 576 and 384. Their common divisor. Their greatest common divisor.

11. What is the smallest number that can be exactly divided by 92, 434, 14, 62, 186, and 1400?

12. How many times is 34×15 contained in $9 \times 7 \times 3 \times 5 \times 2$? 7 times 217 in 62 times 140?

13. Divide 22932 by 1600, using their factors.

14. Divide the least common multiple of 996, 2490, 17430, and 7968 by their greatest common divisor.

15. Find the least common multiple of the first six even numbers. Of the first six odd numbers.

16. What is the largest number that will exactly divide 195 and 225? The smallest number that can be exactly divided by each of them?

17. Find the least number of feet that can exactly contain 26 feet, 312 feet, and 390 feet; and the greatest number that can exactly divide each.

18. Find the product of the least common multiple of 572, 884, 1336, and 2828, by their greatest common divisor.

19. What is the longest chain that will exactly measure the distances 198 feet, 264 feet, and 495 feet?

20. Find the smallest sum of money for which I could buy an exact number of books at \$3, \$5, \$4, or \$6 each.

21. How many days' work at 75 cents a day will pay for 125 bushels of corn at 30 cents a bushel?

22. What is the greatest length of the rails that will exactly lay three side-tracks of a railroad 3013, 2231, and 2047 feet long?

23. What is the least number of acres that can be divided into farms of 150, 200, or 250 acres each?

24. How many tons of hay at \$24 a ton can be exchanged for 16 thousand feet of boards at \$30 a thousand?

25. A boy desires to cut three balls of twine containing 300 feet, 360 feet, and 600 feet into kite-strings of equal length. What is the greatest length he can make them?

Review Questions.

1. What are the ***Factors*** of a number? Define composite number. Explain the difference between a composite and a prime number. Name five composite numbers, and tell what their factors are. Give five examples of prime numbers. Tell the difference between an even number and an odd number. Give five examples of even and of odd numbers.

2. Define ***Exact Divisor.*** Give three examples of exact divisors. What is the only difference between the terms factor and divisor, and show it by an example? What is a prime factor, or divisor? Name three prime and three composite factors of 24. What is ***Factoring?*** State the principles of factoring. Tell how to find the prime factors, or divisors, of a number. Explain how to find all the factors of a number.

3. Define ***Divisor,*** or ***Measure.*** Give three examples. What is a common divisor? Define ***Greatest Common Divisor.*** Name three common divisors, and also the greatest common divisor of 18 and 24. When are numbers prime to each other? State the principles of common divisors. Tell how to find a common divisor. Tell how to find a common divisor, and how to find all the common divisors of two or more numbers. Repeat the rule to find the greatest common divisor of two numbers; and of more than two numbers.

4. Define ***Multiple,*** or ***Dividend.*** Give two examples. What is a common multiple? Define ***Least Common Multiple.*** Name three common multiples of 4 and 6; and their least common multiple. Tell the difference between multiple and dividend. State the principles of common multiples. Tell how to find the multiple of a number. How to find a common multiple. What is the rule to find the least common multiple of two or more numbers?

5. Define ***Cancellation.*** What is the chief use of cancellation? What effect upon a number is produced by omitting one of the factors of that number? If a factor is omitted from the divisor only, is the quotient made greater, or less? What effect upon the quotient is produced by striking out a factor from the dividend only? What is the effect upon the quotient if a common factor is rejected from both dividend and divisor? State the principles of cancellation. Repeat the rule for cancellation.

SECTION VIII.

FRACTIONS.

1. If an apple is cut into two equal parts, what is each part called? What part of the apple is each piece?

2. Into how many halves can an apple be cut? How many halves are in a unit, or in any thing?

3. If an apple is cut into four equal parts, what is each part called? What are three of the pieces called?

4. Into how many fourths can an apple be cut? How many fourths are in a unit, or in any thing?

5. If a cake is cut into three equal parts, what is each part called? What are two of the parts? Three?

6. Into how many thirds can a cake be cut? How many thirds are in a unit, or in any thing?

7. If a cake is cut into six equal parts, what part of the cake is in each piece? In three of the parts?

8. How many sixths are in a cake? How many sixths are in a unit, or in any thing?

9. What is one of five equal parts of a melon called? What are two parts? How many fifths in a unit?

10. Into how many halves can a unit, or any thing, be divided? Into how many thirds? Fourths? Fifths? Sixths?

11. What is meant by one-half of a unit? By one-third? One-fourth? One-fifth? One-sixth?

12. What is meant by two-thirds of a unit? By three-fourths? Two-fifths? Four-fifths? Three-sixths? Five-sixths? Four-sevenths? Six-sevenths?

13. What part of a unit is one of two equal parts of the unit? One of three equal parts? Two of four equal parts? Three of five equal parts?

14. Which is the greater, one-half or one-fourth of an apple? One-third, or one-sixth? One-fourth, or one-eighth? One-fifth, or one-sixth?

15. Which are the smaller parts of any thing, halves or fourths? Thirds, or fourths? Fourths, or fifths? Fifths, or sixths? Sevenths, or eighths?

Definitions.

201. A ***Fraction*** is one or more of the equal parts of a unit.

Thus, *1 third, 3 fifths, 5 sixths* are fractions.

Fractions are divided into two classes: *Common Fractions* and *Decimal Fractions.*

A ***Common Fraction*** is expressed in figures by two numbers, written one above the other, with a line between them.

One half is expressed	$\frac{1}{2}$.	Three sixths is expressed	$\frac{3}{6}$.
One third "	$\frac{1}{3}$.	Four ninths "	$\frac{4}{9}$.
Two thirds "	$\frac{2}{3}$.	Six tenths "	$\frac{6}{10}$.
Three fourths "	$\frac{3}{4}$.	Ten fortieths "	$\frac{10}{40}$.

202. The two numbers used to express a fraction are called the *Denominator* and the *Numerator*.

203. The ***Denominator*** of a fraction is the number written *below* the line.

The denominator shows into *how many equal parts* the unit is divided, and *denominates*, or *gives name to*, the parts.

Thus, *10* is the denominator of the fraction $\frac{9}{10}$; it shows that the unit is divided into ten equal parts, which are named *tenths*.

204. The ***Numerator*** of a fraction is the number written *above* the line.

The numerator shows the *number* of the equal parts of the unit expressed by the fraction.

Thus, *9* is the numerator of the fraction $\frac{9}{10}$; it shows that 9 of ten equal parts of a unit are expressed by the fraction.

205. The ***Terms*** of a fraction are its numerator and denominator.

Thus, *9* and *10* are the terms of the fraction $\frac{9}{10}$.

206. A ***Fractional Unit*** is one of the equal parts into which the unit is divided.

Thus, $\frac{1}{3}$ is the fractional unit of $\frac{2}{3}$; $\frac{1}{5}$ of $\frac{3}{5}$; \$$\frac{1}{4}$ of \$$\frac{3}{4}$.

Note. — The value of a fractional unit depends upon the number of equal parts into which the integral unit is divided.

Thus, if an apple is divided into *halves*, each part is larger than it would be if the apple were divided into *fourths*, or into *eighths*.

207. Principles.

I. *The less the number of equal parts into which the unit is divided, the greater is the value of each of the parts.*

II. *The greater the number of equal parts into which the unit is divided, the less is the value of each of the parts.*

Notation and Numeration of Fractions.

208. A ***common fraction is written*** by placing the numerator above the denominator, with a line drawn between them.

Thus, *six sevenths* is written $\frac{6}{7}$; *seven tenths*, $\frac{7}{10}$.

209. A ***common fraction is read*** by stating the number of parts shown by the numerator, and then giving the name of the parts indicated by the denominator.

Thus, $\frac{8}{9}$ is read *eight ninths*; $\frac{5}{12}$, *five twelfths*; $\frac{123}{456}$, *one hundred and twenty-three four hundred and fifty-sixths.*

Exercises.

Express in figures, or write, the fraction *six ninths.*

210. Model. — The fraction *six ninths* is expressed in figures by writing *6* as the *numerator*, and *9* as the *denominator*, giving $\frac{6}{9}$, the fraction required.

1. Express in figures the following fractions: —

Two fifths; three eighths; four sevenths; five eighths; three ninths; six tenths; two elevenths; ten twelfths; nine fifteenths; sixteen twentieths.

Eleven thirtieths; fourteen thirty-fifths; eighteen forty-thirds; twenty fiftieths; twenty-five sixty-thirds; thirty-one seventieths; one hundred one hundred and sixths; two hundred two hundredths; one hundred and five two hundred and fifty-seconds; two hundred and fifty three hundred and sevenths; three hundred and forty-nine four hundred and seventy-firsts.

Express in words, or read, the fraction $\frac{7}{11}$.

211. Model. — The fraction $\frac{7}{11}$ is read by stating the *number* of equal parts shown by the numerator, *seven*, and then the *name* of the equal parts shown by the denominator, *elevenths*, giving *seven elevenths*, the expression required.

2. Express in words the following fractions: —

1. $\frac{3}{7}$; $\frac{7}{8}$; $\frac{2}{5}$; $\frac{5}{6}$; $\frac{5}{9}$; $\frac{6}{4}$; $\frac{2}{3}$.

2. $\frac{4}{10}$; $\frac{5}{11}$; $\frac{8}{4}$; $\frac{5}{13}$; $\frac{9}{15}$; $\frac{6}{17}$.

3. $\frac{3}{18}$; $\frac{9}{16}$; $\frac{11}{17}$; $\frac{15}{19}$; $\frac{21}{23}$; $\frac{19}{24}$.

4. $\frac{15}{21}$; $\frac{16}{33}$; $\frac{18}{45}$; $\frac{21}{57}$; $\frac{32}{69}$; $\frac{39}{70}$.

5. $\frac{45}{72}$; $\frac{54}{83}$; $\frac{63}{84}$; $\frac{32}{95}$; $\frac{81}{96}$; $\frac{99}{100}$.

6. $\frac{90}{112}$; $\frac{81}{125}$; $\frac{110}{136}$; $\frac{115}{147}$; $\frac{120}{197}$.

7. $\frac{135}{210}$; $\frac{167}{315}$; $\frac{175}{425}$; $\frac{212}{536}$; $\frac{325}{647}$.

8. $\frac{223}{327}$; $\frac{314}{475}$; $\frac{416}{487}$; $\frac{514}{523}$; $\frac{546}{575}$.

9. $\frac{357}{535}$; $\frac{451}{636}$; $\frac{563}{748}$; $\frac{674}{859}$; $\frac{711}{900}$.

10. $\frac{456}{712}$; $\frac{567}{823}$; $\frac{678}{934}$; $\frac{789}{1000}$; $\frac{975}{1200}$.

3. In the preceding exercises, what does the denominator of each fraction show and name, and what does the numerator show?

The Value of a Fraction.

1. If an apple is cut into halves, how many halves of an apple are there?

2. Is a half of an apple greater, or less than 1 apple? Why?

3. Is the value of $\$\frac{3}{4}$ greater, or less than \$1? Is $\frac{4}{5}$ greater, or less than 1? $\frac{5}{6}$? $\frac{7}{8}$?

4. Are two halves of an apple greater, or less than 1 apple? Why? Three thirds? Why?

5. Which has the greater value, $\$\frac{4}{4}$ or \$1? $\frac{3}{3}$ of a yard, or 1 yard? $\frac{8}{8}$ of a gallon, or 1 gallon? Why?

6. One half of 2 apples is equal to how many half-apples? To how many whole apples?

7. If 3 apples are divided into halves, how many half-apples are there in all?

8. 3 half-apples are equal to how many apples? Why? 3 half-apples are what part of 3 whole apples?

9. What is the value of 5 half-apples? Of $\frac{7}{2}$ of a dollar? $\frac{9}{3}$ of a yard? $\frac{20}{10}$ of a cake? Of $\frac{25}{5}$? Of $\frac{12}{12}$?

10. Compare the value of each of the following fractions with a unit, or 1: $\frac{2}{3}$, $\frac{5}{5}$, $\frac{7}{6}$, $\frac{1}{12}$, $\frac{24}{12}$, $\frac{15}{4}$.

11. Indicate in the form of a fraction the division of 1 by 2; 6 by 1; 3 by 8; 12 by 4; 15 by 7; 8 by 18; 24 by 12.

12. As an expression of division, what is the value of $\frac{6}{3}$? Of $\frac{3}{8}$? Of $\frac{12}{6}$? Of $\frac{4}{7}$? Of $\frac{15}{5}$? Of $\frac{6}{12}$? Of $\frac{18}{9}$?

212. From the preceding examples, it appears that

I. *The value of a fraction is less than* 1, *when the numerator is less than the denominator.*

Thus, the value of $\frac{2}{4}$ is *less* than 1, because *not all* the equal parts, or fourths, into which the unit, or 1, is divided, are taken.

II. *The value of a fraction is* 1, *when the numerator and the denominator are alike.*

Thus, the value of $\frac{4}{4}$ is *1*, because *all* the equal parts, or fourths, into which the unit, or 1, is divided, are taken.

III. *The value of a fraction is greater than* 1, *when the numerator is greater than the denominator.*

Thus, the value of $\frac{6}{4}$ is *greater* than 1, because *more* equal parts, or fourths, are taken than are in a unit, or 1.

IV. *The value of a fraction depends upon the number of equal parts into which the unit is divided, and the number of equal parts expressed by the fraction.*

Definitions.

213. According to value, fractions are divided into two kinds: *Proper Fractions* and *Improper Fractions.*

214. A ***Proper Fraction*** is a fraction whose value is less than 1.

The numerator of a proper fraction is always less than its denominator.

Thus, $\frac{2}{3}$, $\frac{4}{15}$, and $\frac{20}{25}$ are *proper fractions.*

215. An ***Improper Fraction*** is a fraction whose value is equal to, or greater than 1.

The numerator of an improper fraction is always equal to, or greater than, its denominator.

Thus, $\frac{3}{3}$, $\frac{15}{4}$, and $\frac{25}{20}$ are *improper fractions.*

216. A ***Mixed Number*** is a number expressed by an integer and a fraction.

Thus, $2\frac{1}{2}$ is a *mixed number,* read *2 and* $\frac{1}{2}$; $17\frac{1}{6}$, read *17 and* $\frac{1}{6}$.

217. An ***integer*** may be expressed in the ***form of a fraction*** by writing the integer as the numerator, and 1 as the denominator.

Thus, 4 may be expressed by $\frac{4}{1}$, read 4 *ones.*

218. Fractions may be considered as indicating division; the numerator being the dividend, and the denominator being the divisor.

Thus, $\frac{6}{6}$ is 5 sixths of a unit, or is 5 divided by 6; $\$\frac{40}{5}$ is 40 fifths of \$1, or is \$40 ÷ 5.

Also, 1 divided by $3 = \frac{1}{3}$; $3 \div 4 = \frac{3}{4}$; $14 \div 5 = \frac{14}{5}$.

219. Principle.

The value of a fraction is the quotient of its numerator divided by its denominator.

Exercises.

1. Analyze the fraction $\frac{7}{8}$.

220. Model 1. — $\frac{7}{8}$ is read *seven eighths;* its *terms* are 7 and 8; the *denominator* is 8, which shows that the unit is divided into 8 equal parts; the *numerator* is 7, which shows that 7 of these equal parts are expressed; it is a *proper fraction,* since its numerator is less than its denominator, and its value is less than 1.

2. Analyze the mixed number $7\frac{8}{9}$.

221. Model 2. — $7\frac{8}{9}$ is read *seven and eight ninths;* it is a *mixed number,* since it is a number expressed by a whole number and a fraction; the whole number is *7;* the fraction is $\frac{8}{9}$; the terms of the fraction are 8 and 9; etc. (Model 1).

Analyze the following fractions: —

1. $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{5}{6}$.
2. $\frac{3}{5}$; $\frac{5}{8}$; $\frac{6}{7}$; $\frac{7}{5}$.
3. $\frac{5}{7}$; $\frac{7}{8}$; $\frac{8}{9}$; $\frac{9}{6}$.
4. $\frac{7}{9}$; $\frac{9}{10}$; $\frac{8}{11}$; $\frac{10}{8}$.
5. $\frac{9}{11}$; $\frac{11}{12}$; $\frac{7}{13}$; $\frac{13}{9}$.
6. $\frac{5}{3}$; $\frac{5}{9}$; $\frac{7}{4}$; $\frac{7}{12}$.
7. $\frac{4}{11}$; $\frac{9}{5}$; $\frac{5}{13}$; $\frac{15}{11}$.
8. $\frac{10}{7}$; $\frac{13}{15}$; $\frac{23}{16}$; $\frac{7}{17}$.
9. $\frac{11}{31}$; $\frac{35}{18}$; $\frac{19}{42}$; $\frac{51}{19}$.
10. $\frac{53}{21}$; $\frac{32}{53}$; $\frac{97}{65}$; $\frac{9}{77}$.
11. $7\frac{2}{9}$; $9\frac{3}{11}$.
12. $13\frac{7}{13}$; $25\frac{12}{27}$.
13. $34\frac{21}{35}$; $45\frac{32}{45}$.
14. $56\frac{43}{52}$; $67\frac{57}{68}$.
15. $78\frac{65}{79}$; $89\frac{99}{100}$.

222. Since a fraction indicates the division of the numerator by the denominator, any change in the terms of a fraction must affect the value of the fraction, in the same manner as any change in the divisor and the dividend affects the value of the quotient in division (197), as shown in the following illustrations: —

I. *If the numerator of a fraction is multiplied by any number, the value of the fraction is made greater.*

Thus, if the numerator of $\frac{1}{4}$ is multiplied by 2, the result is $\frac{2}{4}$, or $\frac{1}{4} \times 2 = \frac{2}{4}$, which is 2 times as great as $\frac{1}{4}$.

For, the value of each of the fractional units remains the same, while the number of the units is made 2 times as great.

II. *If the numerator of a fraction is divided by any number, the value of the fraction is made less.*

Thus, if the numerator of $\frac{4}{5}$ is divided by 2, the result is $\frac{2}{5}$, or $\frac{4}{5} \div 2 = \frac{2}{5}$, which is but $\frac{1}{2}$ as great as $\frac{4}{5}$.

For, the value of each of the fractional units remains the same, while the number of the units is made but $\frac{1}{2}$ as great.

III. *If the denominator of a fraction is multiplied by any number, the value of the fraction is made less.*

Thus, if the denominator of $\frac{1}{4}$ is multiplied by 2, the result is $\frac{1}{8}$, or $\frac{1}{4} \times 2 = \frac{1}{8}$, which is but $\frac{1}{2}$ as great as $\frac{1}{4}$.

For, the value of each fractional unit is made but $\frac{1}{2}$ as great, while the number of the units remains the same.

IV. *If the denominator of a fraction is divided by any number, the value of the fraction is made greater.*

Thus, if the denominator of $\frac{1}{8}$ is divided by 2, the result is $\frac{1}{4}$, or $\frac{1}{8} \div 2 = \frac{1}{4}$, which is 2 times as great as $\frac{1}{8}$.

For, the value of each fractional unit is made 2 times as great, while the number of the units remains the same.

V. *If both terms of a fraction are multiplied by the same number, the value of the fraction is not changed.*

Thus, if both terms of $\frac{1}{4}$ are multiplied by 2, the result is $\frac{2}{8}$, or $\frac{1}{4} \times \frac{2}{2} = \frac{2}{8}$, which has the same value as $\frac{1}{4}$.

For, while the value of each fractional unit is made but $\frac{1}{2}$ as great, the number of the units is made 2 times as great.

VI. *If both terms of a fraction are divided by the same number, the value of the fraction is not changed.*

Thus, if both terms of the fraction $\frac{2}{8}$ are divided by 2, the result is $\frac{1}{4}$, or $\frac{2}{8} \div \frac{2}{2} = \frac{1}{4}$, which has the same value as $\frac{2}{8}$.

For, while the value of each fractional unit is made 2 times as great, the number of the units is made but $\frac{1}{2}$ as great.

From the preceding illustrations and explanations are deduced the following

223. General Principles of Fractions.

I. The value of a fraction is multiplied

1. *By multiplying its numerator; or*
2. *By dividing its denominator.*

II. The value of a fraction is divided

1. *By dividing its numerator; or*
2. *By multiplying its denominator.*

III. The value of a fraction is not changed

1. *By multiplying both terms by the same number; or*
2. *By dividing both terms by the same number.*

Review.

1. What is a ***Fraction?*** Into what two classes are fractions divided? How are common fractions expressed? Give five examples. What are the two numbers used to express a fraction called? Define ***Denominator.*** What does it show and name? Give two examples. Define ***Numerator.*** What does it show? Give two examples. What are the terms of a fraction? Give an example.

2. Define ***Fractional Unit.*** Give five examples. Upon what does the value of a fractional unit depend? How is a common fraction written, or notated? Give two examples. How is it read, or numerated? Give two examples. State the principles that govern the value of a fractional unit.

3. When is the value of a fraction less than 1? Why? Give two examples. When is the value equal to 1? Why? Give two examples. When is it greater than 1? Why? Give two examples. Upon what does the value of a fraction depend? According to their value, how many kinds of fractions are there?

4. Define ***Proper Fraction,*** and give three examples. How does the numerator of a proper fraction compare with the denominator? Define ***Improper Fraction,*** and give three examples. How does the numerator of an improper fraction compare with the denominator? Define ***Mixed Number.*** How is a mixed number read? Give two examples. How is an integer expressed in the form of a fraction? As an expression of division, what does the numerator of a fraction indicate? The denominator? Give an example. State the principle that governs the value of a fraction.

5. What effect upon the value of a fraction is produced by multiplying the numerator of a fraction by any number? Explain the reason. By dividing the numerator? Explain. By multiplying the denominator? Explain. By dividing the denominator? Explain. By multiplying both numerator and denominator by the same number? Explain. By dividing both numerator and denominator by the same number? Explain. State the general principles of fractions.

SECTION IX.

REDUCTION OF FRACTIONS.

224. Reduction of Fractions is the process of changing their *form* without changing their *value.*

CASE I.

To Change a Fraction to Lower Terms.

1. If a pear is cut into 4 equal parts, what part of the pear is one of the pieces?

2. If 1 fourth of a pear is cut into 2 equal parts, what part of the pear is one of the pieces?

3. How many fourths are in 6 eighths?

225. Analysis. — Since 1 fourth equals 2 eighths, 6 eighths equal as many fourths as the number of times 2 eighths are contained in 6 eighths, which is 3 times.

Hence, there are 3 fourths in 6 eighths.

4. To how many halves are $\frac{2}{4}$ equal? $\frac{12}{8}$? $\frac{5}{10}$? $\frac{18}{12}$?

5. Express the value of $\frac{12}{24}$ in twelfths; in sixths; in halves.

6. What prime factor is common to the terms of $\frac{18}{24}$? Divide both terms by their common factor.

7. Express the value of $\frac{8}{12}$ in sixths. By what common factor are both terms divided?

8. Express the value of $\frac{24}{36}$ in terms $\frac{1}{12}$ as great; $\frac{1}{6}$ as great; $\frac{1}{4}$ as great; $\frac{1}{3}$ as great; $\frac{1}{2}$ as great.

9. Express in smaller terms the value of $\frac{15}{20}$; $\frac{22}{33}$; $\frac{21}{35}$.

10. Name all the divisors common to the terms of $\frac{27}{36}$. What is their greatest common divisor?

11. What fraction is produced by dividing both terms of $\frac{27}{36}$ by their greatest common divisor?

12. Change to their lowest, or smallest, terms $\frac{18}{32}$; $\frac{25}{35}$; $\frac{28}{42}$; $\frac{32}{48}$; $\frac{36}{54}$; $\frac{48}{60}$; $\frac{56}{72}$; $\frac{65}{78}$; $\frac{72}{84}$.

226. A fraction is reduced to ***lower terms*** when it is changed to a fraction of equal value having a smaller numerator and denominator.

Thus, $\frac{12}{18}$, expressed in lower terms, equals $\frac{6}{9}$ or $\frac{2}{3}$.

227. A fraction is reduced to its ***lowest terms*** when its numerator and denominator are made prime to each other.

Thus, $\frac{8}{9}$ is in its lowest terms, because no integer greater than 1 will exactly divide both *8* and *9*.

228. Principle.

Dividing both terms of a fraction by the same number does not change the value of the fraction.

Written Exercises.

229. *Example.* — Reduce $\frac{225}{250}$ to its lowest terms.

SOLUTION.

$$\frac{225 \div 25}{250 \div 25} = \frac{9}{10}$$

$$225)250(1$$
$$225$$
$$\overline{25)225(9}$$
$$225$$
$$\overline{\quad}$$

EXPLANATION.—The greatest common divisor of the terms of the fraction, 225 and 250, is 25.

Since dividing both terms of a fraction by the same number does not change the value of the fraction, divide the numerator and the denominator by 25, their greatest common factor or divisor, giving $\frac{9}{10}$, which is in its lowest terms, since its terms, 9 and 10, are prime to each other.

230. Rule to Change Fractions to their Lowest Terms.

Divide the terms of the fraction by their greatest common divisor. The resulting fraction will be in its lowest terms.

Note. — A fraction may be reduced to *lower terms* by rejecting any factor common to the terms of the given fraction.

Problems.

Change, or reduce to their lowest terms —

1. $\frac{32}{48}$; $\frac{40}{56}$; $\frac{48}{64}$; $\frac{54}{72}$; $\frac{76}{95}$.
2. $\frac{75}{100}$; $\frac{84}{108}$; $\frac{96}{144}$; $\frac{99}{165}$.
3. $\frac{112}{184}$; $\frac{144}{272}$; $\frac{198}{363}$; $\frac{225}{490}$.
4. $\frac{318}{583}$; $\frac{399}{608}$; $\frac{476}{630}$; $\frac{531}{767}$.
5. $\frac{605}{847}$; $\frac{660}{924}$; $\frac{246}{984}$; $\frac{360}{1008}$.
6. $\frac{105}{1085}$; $\frac{525}{2010}$; $\frac{627}{3439}$; $\frac{870}{4408}$.

7. $\frac{17}{153}$; $\frac{153}{34}$; $7\frac{39}{65}$; $12\frac{52}{143}$; $23\frac{72}{156}$; $34\frac{132}{275}$.

8. $\frac{23}{92}$ of a ton; $\frac{45}{105}$ of a foot; $\frac{65}{320}$ of an acre.

9. \$$5\frac{75}{100}$; $23\frac{275}{1780}$; $45\frac{672}{2240}$ tons; $56\frac{378}{2520}$ gallons.

10. Express in their lowest terms the value of the fractions $\frac{65}{104}$, $\frac{121}{33}$, $\frac{264}{2008}$, $\frac{2700}{243}$, $\frac{1050}{10752}$, $\frac{2226}{19133}$, and $\frac{968}{11616}$.

11. What are the lowest terms in which $\frac{3336}{5004}$, $\frac{4624}{10592}$, $\frac{7625}{16775}$, $\frac{11211}{22459}$, $\frac{21021}{31031}$, and $\frac{11011}{21021}$ can be expressed?

12. Find the lowest terms in which $\frac{36}{1008}$, $\frac{2000}{112}$, $17\frac{180}{589}$, \$$125\frac{95}{100}$, $444\frac{1760}{7480}$, and $200\frac{1008}{1728}$ can be expressed.

13. Which is the greater part of a cake, $\frac{5}{10}$ of it, or $\frac{1}{2}$ of it?

14. Which is the greater sum of money, \$$\frac{15}{75}$, or \$$\frac{230}{1250}$?

15. A farm was so divided that one of two persons received $\frac{20}{160}$ of it, and the other $\frac{29}{232}$ of it. Which had the larger portion?

CASE II.

To Change a Fraction to Higher Terms or to a Given Denominator.

1. If an orange be cut into 4 equal pieces, what part of the orange is each piece?

2. If 1 fourth of an orange is cut into two equal pieces, what part of the orange is each piece?

3. In 3 fourths of an orange are how many eighths?

231. Analysis. — Since 1 is equal to 8 eighths, 1 fourth is one-fourth of 8 eighths, which is 2 eighths; and 3 fourths are 3 times 2 eighths, which are 6 eighths. Hence, $\frac{3}{4}$ of an orange $= \frac{6}{8}$ of an orange.

4. To how many sixths are 2 thirds equal? To how many ninths? How many twelfths? Fifteenths?

5. Express the value of $\frac{3}{4}$ in eighths; in twelfths. In terms 4 times as great; 5 times as great.

6. Which expresses the greater value, $\frac{1}{2}$ or $\frac{1}{4}$? $\frac{2}{4}$ or $\frac{1}{8}$? $\frac{1}{2}$ or $\frac{2}{4}$? $\frac{2}{4}$ or $\frac{4}{8}$?

7. The number of eighths in a unit is how many times the number of halves? How many times the fourths?

8. How is the fraction $\frac{2}{3}$ changed to sixths expressing the same value? To ninths? To twelfths?

9. Name the first three multiples of 4, the denominator of $\frac{1}{4}$. The first three of 5 in $\frac{1}{5}$.

10. Name three fractions of equal value to which $\frac{1}{4}$ can be changed; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{7}$; $\frac{1}{8}$; $\frac{2}{9}$; $\frac{3}{10}$; $\frac{4}{11}$.

11. What fraction is produced by multiplying each term of $\frac{4}{5}$ by 2? By 3? By 4? By 5?

12. Change $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$ to 24ths, to 48ths, and 96ths.

232. A fraction is changed to ***higher terms*** when it is changed to a fraction of equal value having a larger numerator and denominator.

Thus, $\$\frac{1}{2}$ may be expressed in higher terms, as $\$\frac{2}{4}$, $\frac{4}{8}$, $\frac{5}{10}$, etc.

233. A *Given Denominator* is a denominator named to which another is to be changed.

Thus, if $\frac{2}{4}$ is to be changed to eighths, the given denominator is *eighths*, and the equivalent fraction is $\frac{4}{8}$.

234. Principle.

Multiplying both the terms of a fraction by the same number does not change the value of the fraction.

Written Exercises.

235. *Example.* — Change $\frac{4}{9}$ to 108ths.

SOLUTION.

$108 \div 9 = 12$

$\frac{4}{9} \times \frac{12}{12} = \frac{48}{108}$

EXPLANATION.—Since 108, the given denominator, is a multiple of the denominator 9, the other factor required to produce it is the quotient of 108 divided by 9, which is 12.

Since the denominator of $\frac{4}{9}$ must be multiplied by 12, the numerator must also be multiplied by 12, that the value of the fraction may not be changed. Hence, $\frac{4}{9}$ equals $\frac{48}{108}$, the fraction required.

236. Rule to Change a Fraction to Higher Terms or to a Given Denominator.

Divide the given denominator by the denominator of the fraction, and multiply both terms of the fraction by the quotient. The resulting fraction will be in the higher terms required.

Problems.

Reduce, or change —

1. $\frac{2}{3}$ and $\frac{4}{5}$ to fifteenths.
2. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ to twelfths.
3. $\frac{3}{4}$, $\frac{4}{5}$, $\frac{9}{10}$ to twentieths.
4. $\frac{5}{8}$, $\frac{7}{8}$, $\frac{11}{12}$ to 24ths; 48ths.
5. $\frac{8}{9}$, $\frac{7}{12}$, $\frac{17}{18}$ to 36ths; 72nds.
6. $\frac{5}{8}$, $\frac{15}{16}$, $\frac{23}{24}$ to 48ths; 96ths.
7. $\frac{3}{5}$ and $\frac{6}{7}$ to fractions each having 35 as the denominator.
8. $\frac{11}{15}$ and $\frac{8}{25}$ to fractions whose denominator is 75.
9. $\frac{17}{25}$ to 100ths; $\frac{17}{36}$ to 108ths; $\frac{11}{50}$ to 150ths; $\frac{13}{60}$ to 300ths.
10. $\frac{7}{16}$ to 32nds, 48ths, 64ths, 80ths, 96ths, and 112ths.
11. Change $\frac{5}{6}$, 6, $7\frac{1}{10}$, $\frac{11}{20}$, 30, and $\frac{23}{60}$ to sixtieths.
12. Express 7, $5\frac{1}{7}$, $\frac{11}{12}$, 20, $\frac{15}{14}$, $\frac{20}{21}$, 25, and $\frac{1}{84}$ as fractions having 84 as their denominator; having 336.
13. Express the value of $\$\frac{3}{4}$ in hundredths of a dollar.
14. Change $\frac{11}{20}$ of a foot, and $\frac{9}{36}$ of a yard to fractions having terms 3 times, 5 times, and 10 times as great.
15. Reduce $\frac{4}{7}$, $\frac{9}{14}$, $\frac{11}{28}$, 7, $4\frac{3}{56}$, and 112 to 112ths of a ton.

CASE III.

To Change an Integer or a Mixed Number to an Improper Fraction.

1. How many halves of an apple are in 1 apple? How many thirds of an apple? How many fourths?

2. In 2 pears, how many halves of a pear?

237. Analysis. — Since in 1 pear there are 2 halves of a pear, in 2 pears there are 2 times 2 halves, or 4 halves of a pear.

3. In 3 pears, how many halves of a pear? How many thirds? How many fourths? Fifths?

4. How many fifths of a dollar are in \$1? How many in \$2? \$3? \$6? \$9? \$10? \$15?

5. If 4 pints of berries are divided among some boys, to how many could a half-pint each be given?

6. How many fourths of a cake are 2 and 3 fourths cakes?

238. Analysis. — Since in 1 cake there are 4 fourths, in 2 cakes there are 2 times 4 fourths, or 8 fourths; 8 fourths and 3 fourths are 11 fourths. Hence, in $2\frac{3}{4}$ cakes are $\frac{11}{4}$ of a cake.

7. How many sixths are in 2 apples? In $2\frac{1}{6}$ apples? In 3 yards? In $5\frac{1}{6}$ yards? $9\frac{1}{6}$ yards?

8. How many ninths of a dollar are in \$7? In $7\frac{3}{9}$? Tenths of a foot in 8 feet? $8\frac{3}{10}$ feet?

9. How many tenths of an acre in $7\frac{7}{10}$ acres? How many quarter-pounds of candy in $12\frac{3}{4}$ pounds?

10. Change 5, 7, 6, and 10 to eighths, and to tenths; and change $5\frac{1}{5}$, $7\frac{3}{9}$, $9\frac{5}{11}$ to improper fractions.

Written Exercises.

239. *Example* 1. — Change 7 to ninths.

SOLUTION.

9 ninths.
7

63 ninths $= \frac{63}{9}$.

EXPLANATION. — Since in 1 there are 9 ninths, in 7 there are 7 times 9 ninths, which are 63 ninths.

Hence, 7 equals $\frac{63}{9}$.

240. *Example* 2.—Change $20\frac{1}{10}$ to an improper fraction.

SOLUTION.

$20\frac{1}{10} = 20 + \frac{1}{10} = \frac{201}{10}$

10 tenths.
20
200 tenths
1 tenth.
201 tenths $= \frac{201}{10}$

EXPLANATION. — $20\frac{1}{10}$ is equivalent to $20 + \frac{1}{10}$.

Since in 1 there are 10 tenths, in 20 there are 20 times 10 tenths, which are 200 tenths; 200 tenths + 1 tenth are 201 tenths.

Hence, $20\frac{1}{10}$ equals $\frac{201}{10}$, the improper fraction required.

241. Rules to Change an Integer or a Mixed Number to an Improper Fraction.

I. To change an integer to an improper fraction,

Multiply the given denominator by the integer, and under the product write the given denominator (236).

II. To change a mixed number to an improper fraction,

Multiply the denominator of the fraction by the integer, and to the product add the numerator of the fraction.

The denominator written under this result will form the improper fraction required.

Problems.

Change, or reduce —

1. 7 and 9 to thirds.
2. 8 and 10 to fifths.
3. 9 and 12 to sixths.
4. 10 and 15 to tenths.
5. $7\frac{3}{4}$ and $11\frac{3}{4}$ to fourths.
6. $8\frac{1}{8}$ and $21\frac{7}{8}$ to eighths.
7. $9\frac{7}{9}$ and $76\frac{8}{9}$ to ninths.
8. $10\frac{1}{12}$ and $98\frac{7}{12}$ to twelfths.
9. 20 quarts to fourths of a quart; \$30 to half-dollars.
10. 25 gallons to eighths of a gallon; $96\frac{7}{12}$ years to twelfths of a year; $100\frac{1}{200}$ tons to 200ths of a ton.

Change, or reduce, to improper fractions —

11. $12\frac{1}{3}$; $25\frac{7}{25}$; $40\frac{11}{48}$; $50\frac{1}{50}$; $60\frac{1}{60}$; $78\frac{5}{32}$; $89\frac{15}{43}$; $98\frac{17}{54}$.

12. $100\frac{1}{10}$; $200\frac{1}{20}$; $300\frac{1}{40}$; $400\frac{11}{50}$; $500\frac{99}{500}$; $600\frac{111}{800}$.

13. $125\frac{7}{16}$; $236\frac{9}{27}$; $347\frac{11}{38}$; $458\frac{13}{49}$; $569\frac{9}{69}$; $780\frac{11}{70}$.

14. $1615\frac{1}{125}$; $1725\frac{11}{115}$; $1870\frac{5}{135}$; $1908\frac{10}{173}$; $2000\frac{1}{100}$.

15. Express 25 in the form of an improper fraction having 20 as its denominator; having 25; 50; 100.

16. Change 39 to a fraction whose denominator shall be 39; change 55 to fifty-fifths; 100 to hundredths.

17. How many hundredths of a dollar are \$50? $\$75\frac{25}{100}$? $\$1000\frac{99}{100}$? $\$2000\frac{75}{100}$? $\$3500\frac{1}{10}$? $\$5050\frac{1}{50}$?

18. How many quarter-pounds of candy are in a package weighing $12\frac{3}{4}$ pounds? How many eighths of a pound?

CASE IV.

To Change an Improper Fraction to an Integer or to a Mixed Number.

1. How many thirds of a cake are in 1 cake? How many quarter-dollars are in 1 dollar?

2. How many days are in $\frac{7}{2}$ of a day?

242. Analysis. — Since in 1 day there are 2 half-days, in 7 half-days there are as many days as the number of times 2 half-days are contained in 7 half-days, which is $3\frac{1}{2}$ times. Hence, etc.

3. How many pears are in $\frac{8}{4}$ of a pear? In $\frac{12}{6}$?

4. How many ones in $\frac{10}{5}$? In $\frac{14}{7}$? In $\frac{20}{10}$? In $\frac{12}{6}$?

5. A boy bought 8 half-pint boxes of berries. How many pints did he buy?

6. Change $\frac{24}{12}$ of a dozen to dozens; $\$\frac{100}{12}$ to dollars.

7. How many yards are $\frac{15}{3}$ of a yard? $\frac{17}{3}$ of a yard? $\frac{36}{12}$ of a yard? $\frac{99}{12}$ of a yard? $\frac{125}{36}$ of a yard?

8. To what integer or mixed number is $\frac{15}{5}$ equal? $\frac{25}{10}$? $\frac{40}{20}$? $\frac{55}{30}$? $\frac{19}{7}$? $\frac{28}{9}$? $\frac{47}{11}$? $\frac{65}{13}$? $\frac{78}{14}$? $\frac{80}{20}$? $\frac{96}{25}$?

9. How many pounds of tea in 35 quarter-pounds?

10. If I buy $4\frac{1}{8}$ yards of cloth for a suit of clothes, how many yards do I buy?

Written Exercises.

243. *Example.*—Change $3\frac{7}{2}\frac{1}{0}$ to an integer or to a mixed number.

SOLUTION.

$\frac{371}{20} = 371 \div 20 = 18\frac{11}{20}$

$$
\begin{array}{r|l}
20 & 371\,(18\frac{11}{20} \\
\hline
 & 20 \\
 & \hline
 & 171 \\
 & 160 \\
 & \hline
 & 11
\end{array}
$$

EXPLANATION.—Since twenty twentieths equal 1, 371 twentieths equal as many ones as the number of times 20 twentieths are contained in 371 twentieths, or 20 in 371, which is $18\frac{11}{20}$ times.

Hence, $\frac{371}{20} = 18\frac{11}{20}$, the mixed number required.

244. Rule to Change an Improper Fraction to an Integer or to a Mixed Number.

Divide the numerator by the denominator. The quotient will be the integer or the mixed number required.

Problems.

Change, or reduce —

1. $\frac{8}{4}$ of a yard to yards.
2. $\frac{18}{6}$ of a ton to tons.
3. $\frac{40}{8}$ of a foot to feet.
4. $\frac{24}{7}$ and $\frac{48}{14}$ of a week to weeks.
5. $\frac{37}{10}$ and $\frac{99}{20}$ of a mile to miles.
6. $\frac{40}{20}$, $\frac{61}{20}$, and $\frac{89}{30}$ to ones.
7. $\frac{32}{16}$, $\frac{48}{16}$, $\frac{65}{16}$, $\frac{64}{32}$, $\frac{75}{32}$, and $\frac{200}{10}$ of a pound of tea to pounds.
8. $\frac{100}{25}$ of a dollar, $\$\frac{400}{50}$, and $\$\frac{100}{8}$ to dollars.

Change to integers or mixed numbers —

9. $\frac{21}{9}$; $\frac{32}{11}$; $\frac{43}{15}$; $\frac{54}{27}$; $\frac{98}{25}$; $\frac{100}{17}$; $\frac{125}{10}$; $\frac{137}{30}$; $\frac{185}{18}$; $\frac{190}{18}$; $\frac{198}{19}$.
10. $\frac{200}{10}$; $\frac{171}{17}$; $\frac{352}{21}$; $\frac{425}{25}$; $\frac{576}{38}$; $\frac{698}{45}$; $\frac{708}{50}$; $\frac{875}{75}$; $\frac{909}{100}$.
11. $\frac{300}{15}$; $\frac{401}{20}$; $\frac{503}{25}$; $\frac{717}{35}$; $\frac{829}{41}$; $\frac{1001}{50}$; $\frac{1015}{30}$; $\frac{2010}{95}$; $\frac{3040}{101}$.
12. $\frac{2005}{27}$; $\frac{3010}{39}$; $\frac{4200}{41}$; $\frac{5000}{73}$; $\frac{6007}{100}$; $\frac{7080}{101}$; $\frac{8009}{200}$; $\frac{9098}{310}$.

13. $\frac{10008}{225}$; $\frac{12376}{341}$; $\frac{14887}{452}$; $\frac{15001}{500}$; $\frac{17005}{605}$; $\frac{19025}{701}$.

14. How many gallons are $\frac{501}{25}$, $\frac{2007}{50}$, and $\frac{10000}{101}$ gallons?

15. Express the value $\frac{300}{20}$ of a ton in whole tons.

16. How many dollars are needed to pay a half-dollar to each of 15 boys? To each of 42 boys? 50 boys?

17. How many gallons of oil in 112 cans, each containing $\frac{1}{2}$ of a gallon? In 250 cans? In 378? In 425?

18. What number of acres is in a piece of ground composed of 75 lots, each $\frac{1}{8}$ of an acre? In 100 lots?

CASE V.

To Change Dissimilar Fractions to Similar Fractions.

1. How many fourths in 1? In $\frac{1}{2}$? In $\frac{6}{8}$?

2. Change $\frac{1}{2}$ and $\frac{6}{8}$ to fourths.

245. Analysis. — $\frac{1}{2}$ changed to fourths equals $\frac{2}{4}$; and $\frac{6}{8}$ changed to fourths equals $\frac{3}{4}$. Hence, $\frac{1}{2}$ equals $\frac{2}{4}$, and $\frac{6}{8}$ equals $\frac{3}{4}$.

3. Express $\frac{3}{2}$ and $\frac{3}{5}$ as tenths; $\frac{2}{3}$ and $\frac{5}{6}$ as twelfths.

4. Change $\frac{4}{5}$ and $\frac{7}{10}$ to twentieths; $\frac{5}{6}$ and $\frac{7}{8}$ to 24ths.

5. How is each of the fractions $\frac{1}{3}$ and $\frac{3}{4}$ changed to 12ths? Is the value changed? Why?

6. Change $\frac{2}{3}$ and $\frac{4}{5}$ to fractions having the same denominator; $\frac{5}{6}$ and $\frac{6}{7}$; $\frac{7}{8}$ and $\frac{8}{9}$; $\frac{8}{9}$ and $\frac{9}{10}$.

7. What common denominator can $\frac{6}{7}$ and $\frac{7}{8}$ have, and each still express the same value?

8. What is the common multiple of 4 and 5? Of 5 and 6, the denominators of $\frac{4}{5}$ and $\frac{5}{6}$?

9. Change $\frac{5}{6}$ and $\frac{8}{9}$ to fractions which shall have as a common denominator the common multiple of 6 and 9.

10. What is the least common multiple of 4, 5, and 6? Of the denominators of $\frac{5}{8}$, $\frac{6}{7}$, $\frac{7}{8}$?

11. What is the least number of which the denominators of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ are factors?

12. What is the least common multiple of the denominators of $\frac{1}{6}$, $\frac{2}{7}$, $\frac{3}{8}$? Change them to fractions of equal value having the least common denominator.

Definitions.

246. *Similar Fractions* are fractions whose denominators are alike.

Thus, $\frac{8}{10}$, $1\frac{7}{10}$, $\frac{17}{10}$ are *similar fractions.*

247. *Dissimilar Fractions* are fractions whose denominators are not alike.

Thus, $\frac{2}{3}$, $1\frac{7}{10}$, $\frac{10}{17}$ are *dissimilar fractions.*

248. A *Common Denominator* is a denominator common to two or more fractions.

Thus, *6* is the common denominator of $\frac{1}{6}$, $\frac{3}{6}$, and $\frac{5}{6}$.

249. The *Least Common Denominator* is the least, or smallest, denominator common to two or more fractions.

Thus, *12* is the least common denominator of $\frac{2}{3}$, $\frac{11}{12}$, $\frac{16}{24}$.

250. Principles.

I. *A common denominator of two or more fractions is a common multiple of their denominators.*

II. *The least common denominator of two or more fractions is the least common multiple of their denominators.*

III. *Multiplying or dividing both terms by the same number does not change the value of the fraction.*

Written Exercises.

251. *Example* 1. — Change $\frac{3}{5}$ and $\frac{5}{7}$ to similar fractions.

SOLUTION.

$5 \times 7 = 35$ ***L. c. mult.***

$35 \div 5 = 7$, and $\frac{3}{5} \times \frac{7}{7} = \frac{21}{35}$

$35 \div 7 = 5$, and $\frac{5}{7} \times \frac{5}{5} = \frac{25}{35}$

EXPLANATION.—The product of the denominators, 5 and 7, is the common multiple, or common denominator of the denominators.

But since the value of the fraction is not to be changed, the denominator and the numerator must be multiplied by the same number.

To change $\frac{3}{5}$ to 35ths, the denominator and the numerator must be multiplied by 7, giving $\frac{21}{35}$.

To change $\frac{5}{7}$ to 35ths, the denominator and the numerator must be multiplied by 5, giving $\frac{25}{35}$.

Hence, $\frac{3}{5}$ and $\frac{5}{7}$, changed to similar fractions, equal $\frac{21}{35}$ and $\frac{25}{35}$.

Problems.

Change to similar fractions —

1. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
2. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$.
3. $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$.
4. $\frac{3}{5}$, $\frac{3}{6}$, $\frac{3}{7}$; $\frac{2}{9}$, $\frac{1}{2}$, $\frac{5}{8}$.
5. $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$; $\frac{1}{3}$, $\frac{6}{7}$, $\frac{1}{4}$.
6. $\frac{4}{6}$, $\frac{4}{9}$, $\frac{4}{8}$; $\frac{7}{8}$, $\frac{2}{5}$, $\frac{6}{7}$.
7. $\frac{5}{7}$, $\frac{7}{9}$, $\frac{9}{11}$; $\frac{2}{3}$, $\frac{3}{10}$, $\frac{4}{5}$, $\frac{5}{2}$.
8. $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$; $\frac{3}{7}$, $\frac{5}{9}$, $\frac{7}{11}$, $\frac{9}{3}$.
9. $\frac{9}{10}$, $\frac{10}{11}$, $\frac{11}{12}$; $\frac{4}{9}$, $\frac{5}{8}$, $\frac{8}{7}$, $\frac{9}{20}$.

10. To what similar fractions are $\frac{3}{7}$, $\frac{5}{9}$, $\frac{7}{12}$ equal?

11. Change to equivalent fractions having a common denominator $\frac{5}{6}$ of a foot and $\frac{7}{8}$ of a foot; \$$\frac{3}{4}$ and \$$\frac{7}{8}$.

12. Change to similar fractions $\frac{1}{2}$ of an acre, $\frac{5}{6}$ of an acre, $\frac{9}{10}$ of an acre, $\frac{3}{4}$ of an acre.

252. *Example* 2. — Change $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$ to similar fractions having the least common denominator.

SOLUTION.

2)*3*, *6*, *8* ***Denominators.***

3)*3*, *3*, *4*

1, *1*, *4*

$2 \times 3 \times 4 = 24$ ***L. c. den.***

$24 \div 3 = 8$, and $\frac{2}{3} \times \frac{8}{8} = \frac{16}{24}$

$24 \div 6 = 4$, and $\frac{5}{6} \times \frac{4}{4} = \frac{20}{24}$

$24 \div 8 = 3$, and $\frac{7}{8} \times \frac{3}{3} = \frac{21}{24}$

$\frac{2}{3}, \frac{5}{6}, \frac{7}{8} = \frac{16}{24}, \frac{20}{24}, \frac{21}{24}$.

EXPLANATION. — The least common denominator, or least common multiple of the denominators 3, 6, and 8, is 24.

But since the value, etc. (251).

To change $\frac{2}{3}$ to 24ths, multiply both terms, etc.

To change $\frac{5}{6}$ to 24ths, multiply both terms, etc.

To change $\frac{7}{8}$ to 24ths, multiply both terms, etc.

Hence, $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$, changed to least similar fractions, are $\frac{16}{24}$, $\frac{20}{24}$, and $\frac{21}{24}$.

253. Rule to Change Dissimilar Fractions to Similar Fractions.

I. *Find the least common multiple of the denominators of the given fractions, for the least common denominator.*

II. *Divide the least common denominator by the denominator of each fraction, and multiply both terms of the fraction by the quotient. The resulting fractions will be the least similar fractions required.*

Change to similar fractions having the least common denominator —

13. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$; $\frac{1}{6}$, $\frac{3}{9}$, $\frac{5}{12}$.

14. $\frac{2}{5}$, $\frac{1}{6}$, $\frac{8}{9}$; $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{16}$.

15. $\frac{3}{5}$, $\frac{5}{8}$, $\frac{9}{16}$; $\frac{5}{6}$, $\frac{7}{9}$, $\frac{8}{15}$.

16. $\frac{7}{10}$, $\frac{5}{12}$, $\frac{8}{15}$.

17. $\frac{7}{12}$, $\frac{7}{15}$, $\frac{7}{18}$.

18. $\frac{9}{20}$, $\frac{7}{24}$, $\frac{7}{30}$.

19. $\frac{11}{36}$, $\frac{11}{42}$, $\frac{11}{48}$.

20. $\frac{12}{45}$, $\frac{13}{60}$, $\frac{14}{75}$.

21. $\frac{25}{48}$, $\frac{37}{80}$, $\frac{49}{96}$.

22. $\frac{2}{3}$ of a mile, $\frac{5}{9}$ of a mile, and $\frac{11}{45}$ of a mile.

23. $\frac{5}{6}$ of a year, $\frac{7}{12}$ of a year, $\frac{9}{13}$ of a year, and $\frac{11}{52}$ of a year.

24. Change to least similar fractions $\frac{5}{8}$, $\frac{11}{48}$, $\frac{17}{72}$, $\frac{22}{96}$.

25. Find the least similar fractions equal to $\frac{11}{26}$, $\frac{16}{39}$, $\frac{25}{52}$.

26. Change $\frac{7}{11}$, $\frac{13}{3}$, $\frac{8}{33}$, $1\frac{9}{44}$, $\frac{16}{121}$ to least similar fractions.

27. What least similar fractions are equal to $\frac{19}{5}$, $\frac{11}{35}$, $\frac{18}{49}$, and $1\frac{5}{7}$? To $\frac{13}{10}$, $1\frac{1}{15}$, $\frac{11}{60}$, $\frac{74}{75}$? $\frac{11}{14}$, $1\frac{1}{35}$, $\frac{13}{42}$, $2\frac{1}{70}$?

28. To what least similar fractions may the dissimilar fractions $\$\frac{5}{7}$, $\$\frac{11}{63}$, $\$\frac{26}{21}$, and $\$\frac{35}{84}$ be changed?

29. Change to equivalent fractions having the least common denominator $\frac{5}{17}$, $\frac{37}{51}$, $\frac{8}{34}$, and $1\frac{1}{68}$.

30. Reduce $\frac{1}{49}$, $\frac{9}{14}$, $1\frac{1}{7}$, $\frac{11}{98}$, and $\frac{5}{2}$ to similar fractions having the least common denominator.

Review Problems.

1. What are the lowest terms to which $\frac{39}{234}$, $\frac{740}{1110}$, $\frac{672}{3024}$, $\frac{702}{2886}$, $\frac{2163}{12463}$, $\frac{2432}{16281}$, $\frac{3525}{25351}$, and $\frac{4230}{35391}$ can be reduced?

2. Change to improper fractions $99\frac{1}{99}$, $100\frac{1}{100}$, $444\frac{4}{111}$.

3. Reduce $\frac{10000}{99}$, $\frac{9999}{100}$, $\frac{5555}{111}$, $\frac{60006}{777}$, $\frac{77777}{999}$, $\frac{80808}{707}$, $\frac{90000}{888}$, and $\frac{100000}{80008}$ to integers or mixed numbers.

4. Change 25 to the form of a fraction having 25 as its denominator. Change it to ones, to 8ths, to 100ths, to 1000ths.

5. Express 7, $\frac{2}{3}$, $3\frac{4}{5}$, $\frac{17}{15}$ as 30ths, as 60ths, as 75ths, as 225ths.

6. Reduce to similar fractions $7\frac{1}{2}$, $\frac{9}{8}$, $11\frac{1}{11}$; $2\frac{1}{7}$, $\frac{8}{9}$, $10\frac{1}{10}$.

7. Change to least similar fractions $\frac{1}{14}$, $5\frac{1}{5}$, $\frac{36}{35}$, and $70\frac{1}{70}$.

8. Ten thirteenths equal how many one hundred forty-thirds? How many two hundred forty-sevenths?

9. What improper fractions equal $505\frac{5}{101}$? $700\frac{1}{70}$? $2000\frac{5}{303}$? $7000\frac{1}{700}$? $8080\frac{1}{808}$? $9009\frac{1}{909}$? $10090\frac{100}{123}$?

10. How many 200ths of a ton are in $7\frac{3}{40}$ and $25\frac{3}{25}$ tons?

11. Express each of the integers 1, 3, 5, 11, 25, 100 as a fraction having as its denominator 1, 3, 5, 11, 25, and 100.

12. In $100\frac{42}{63}$ hogsheads, how many 63rds of a hogshead? How many 252nds? How many 9ths? How many 21sts?

13. Arrange four 5's so that their value shall be 56.

14. Change as many as possible of the following fractions to 60ths: $\frac{3}{5}$, $\frac{7}{12}$, $\frac{5}{24}$, $\frac{19}{18}$, and $1\frac{1}{30}$; to 72nds; all to 360ths.

15. Which is the largest of four lots containing respectively $\frac{21}{28}$, $\frac{60}{80}$, $\frac{9}{12}$, and $\frac{120}{160}$ of an acre?

16. One day a fruit-dealer sold 75 baskets of apples, each containing $\frac{3}{8}$ of a bushel. How many bushels did he sell?

17. How many quarter-pounds are in a piece of walnut-candy weighing $9\frac{3}{4}$ pounds? $12\frac{2}{8}$ pounds? $16\frac{1}{2}$ pounds?

18. Change \$5, $\$\frac{1}{5}$, $\$3\frac{1}{4}$, $\$\frac{11}{10}$, $\$9\frac{99}{100}$ to hundredths.

19. What are the least similar fractions to which $\frac{7}{59}$, $\frac{8}{177}$, $11\frac{1}{2}\frac{1}{3}\frac{1}{6}$, 8, and $\frac{119}{118}$ can be changed?

20. What are the least similar fractions to which $\frac{1}{5}$, $5\frac{1}{10}$, $10\frac{1}{50}$, $5\frac{1}{100}$, and $100\frac{5}{11}$ can be changed?

Review Questions.

1. What is meant by ***Reduction of Fractions?*** What are the ***terms*** of a fraction? How is a fraction reduced to ***lower terms?*** When is it in its lowest terms? How is a fraction changed to its lowest terms? Illustrate by an example. State the principle. Repeat the rule to change a fraction to its lowest terms.

2. When is a fraction changed to ***higher terms?*** What is meant by a given denominator? How is a fraction changed to higher terms? Illustrate by an example. State the principle. Repeat the rule to change a fraction to higher terms or to a given denominator.

3. What is an ***Integer?*** A ***Fraction?*** How is an integer changed to a fraction? Give an example. Repeat the rule. Repeat the rule to change a mixed number to an improper fraction.

4. Define ***Improper Fraction.*** What is a mixed number? Tell how to change an improper fraction to an integer or a mixed number. Give an example, and explain it. Repeat the rule to change an improper fraction to a mixed number or an integer.

5. Define ***Similar Fractions.*** What are dissimilar fractions? What is meant by a common denominator? By the least common denominator? How are dissimilar fractions changed to similar fractions? Illustrate by an example. State the principles. Repeat the rule to change dissimilar fractions to least similar fractions.

SECTION X.

ADDITION OF FRACTIONS.

1. What is the unit of 2 cents? Of 3 cents? What is the sum of 2 cents and 3 cents? Of 2 feet and 3 feet?

2. What is the unit of 2 fourths? Of 3 fourths? What is the sum of 2 fourths and 3 fourths? Of 2 fifths and 3 fifths?

3. If Frank has 2 fourths of an apple, and Edgar has 3 fourths, how many fourths have both?

4. How many ninths are 4 ninths and 7 ninths? $\frac{5}{9} + \frac{8}{9}$?

5. What is the sum of $\frac{3}{5}$ and $\frac{4}{5}$? Of $\frac{1}{8} + \frac{2}{8} + \frac{5}{8}$?

6. What is the numerator of the sum of $\frac{2}{9} + \frac{4}{9} + \frac{8}{9}$? What is the denominator?

7. How do you add similar fractions, or fractions having a common denominator?

8. What is the unit of 2 cents? Of 3 feet? Can 2 cents and 3 feet be united in one sum? Why?

9. What is the unit of 2 thirds? Of 3 fourths? Can 2 thirds and 3 fourths be united in one sum? Why?

10. To what similar fractions can $\frac{2}{3}$ and $\frac{3}{4}$ be changed? Change them to similar fractions, and find their sum.

11. To what kind of fractions must $\frac{3}{4}$ and $\frac{5}{6}$ be changed before they can be added? What least similar fractions do they equal? What is their sum?

12. Harry paid $\$\frac{2}{3}$ for a knife, and $\$\frac{3}{4}$ for a box of paints. How much did he pay for both?

254. Analysis. — Since Harry paid $\$\frac{2}{3}$ for a knife, and $\$\frac{3}{4}$ for a box of paints, he paid for both the sum of $\$\frac{2}{3}$ and $\$\frac{3}{4}$.

$\frac{2}{3}$ changed to 12ths equals $\frac{8}{12}$; and $\frac{3}{4}$ changed to 12ths equals $\frac{9}{12}$. Now $\frac{8}{12}$ and $\frac{9}{12}$ are $\frac{17}{12}$, or $1\frac{5}{12}$. Hence, Harry paid for both $\$1\frac{5}{12}$.

13. What is the sum of $\frac{2}{5}$ of a ton and $\frac{5}{6}$ of a ton?

14. How do you add dissimilar fractions, or fractions having different denominators?

255. Principles.

I. *Only fractional units of the same kind of things can be added.*

II. *Only similar fractions can be added.*

Written Exercises.

256. *Example.* — Find the sum of $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$.

SOLUTION.

$2)4,\ 6,\ 8$ ***Denominators.***

$2)2,\ 3,\ 4$

$1,\ 3,\ 2$

$2 \times 2 \times 3 \times 2 = 24$ ***L. c. d.***

$24 \div 4 = 6$, and $\frac{3}{4} \times \frac{6}{6} = \frac{18}{24}$

$24 \div 6 = 4$, and $\frac{5}{6} \times \frac{4}{4} = \frac{20}{24}$

$24 \div 8 = 3$, and $\frac{7}{8} \times \frac{3}{3} = \frac{21}{24}$

$\frac{3}{4} + \frac{5}{6} + \frac{7}{8} = \frac{18}{24} + \frac{20}{24} + \frac{21}{24} = \frac{18 + 20 + 21}{24} = \frac{59}{24} = 2\frac{11}{24}$.

EXPLANATION.—Since the fractions are dissimilar, they must first be changed to similar fractions having the least common denominator.

The least common multiple of the denominators is 24.

$\frac{3}{4}$ changed to 24ths $= \frac{18}{24}$; $\frac{5}{6} = \frac{20}{24}$; and $\frac{7}{8} = \frac{21}{24}$.

Now the sum of the similar fraction is the number of 24ths expressed by all the fractions, and is found by adding together their numerators 18, 20, and 21; and $\frac{18 + 20 + 21}{24} = \frac{59}{24} = 2\frac{11}{24}$, the sum required.

Problems.

Find the sum of—

1. $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$; $\frac{2}{5}, \frac{3}{5}, \frac{4}{5}$.
2. $\frac{1}{6}, \frac{4}{6}, \frac{5}{6}$; $\frac{2}{7}, \frac{4}{7}, \frac{6}{7}$.
3. $\frac{3}{8}, \frac{5}{8}, \frac{7}{8}$; $\frac{5}{9}, \frac{7}{9}, \frac{8}{9}$.
4. $\frac{5}{10}, \frac{7}{10}, \frac{9}{10}, \frac{11}{10}$.
5. $\frac{5}{6}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}$.
6. $\frac{3}{10}, \frac{5}{12}, \frac{8}{15}$.
7. $\frac{11}{12}, \frac{14}{15}, \frac{17}{18}$.
8. $\frac{13}{15}, \frac{17}{20}, \frac{22}{25}$.
9. $\frac{17}{24}, \frac{29}{30}, \frac{31}{36}$.
10. $\frac{19}{34}, \frac{25}{51}, \frac{33}{68}$.
11. $\frac{23}{42}, \frac{35}{63}, \frac{47}{84}$.
12. $\frac{35}{57}, \frac{49}{76}, \frac{77}{95}$.
13. $\frac{7}{9}, \frac{71}{108}, \frac{8}{27}$; $\frac{5}{13}, \frac{7}{143}, \frac{7}{11}$; $\frac{3}{19}, \frac{17}{57}, \frac{2}{3}, \frac{144}{171}$.
14. $\frac{39}{17}, \frac{3}{4}, \frac{1}{136}$; $\frac{2}{121}, \frac{13}{11}, \frac{7}{33}, \frac{9}{242}$; $\frac{8}{25}, \frac{9}{7}, \frac{3}{175}, \frac{8}{5}$.
15. $\$\frac{3}{4}, \$\frac{7}{10}, \$\frac{1}{2}, \$\frac{17}{25}$; $\frac{5}{9}$ of a yard, $\frac{7}{12}$ of a yard, $\frac{1}{18}$ of a yard.
16. $\frac{7}{112}$ of a ton, $\frac{17}{28}$ of a ton; $\frac{7}{360}$ of a year, $\frac{7}{12}$ of a year.
17. Add $\frac{5}{19}, \frac{7}{5}, \frac{8}{95}, \frac{11}{76}$; $\frac{7}{108}, \frac{17}{81}, \frac{1}{4}, \frac{11}{6}$; $\frac{3}{26}, \frac{4}{13}, \frac{5}{39}, 1\frac{1}{52}$.
18. Find the sum of $\frac{1}{17}, \frac{52}{51}$, and $\frac{4}{119}$; of $\frac{1}{252}, \frac{19}{18}$, and $\frac{11}{72}$.
19. What is the amount of $\frac{1}{11}, \frac{188}{187}, \frac{16}{17}$? Of $\frac{3}{13}, \frac{11}{585}, \frac{180}{117}$?
20. How many cords are $\frac{15}{16}$ of a cord + $\frac{9}{8}$ of a cord + $\frac{117}{128}$ of a cord? $\frac{3}{4}$ of a cord, $\frac{5}{16}$ of a cord, and $\frac{9}{32}$ of a cord?
21. What part of a dollar is the sum of $\$\frac{5}{10} + \$\frac{6}{100} + \$\frac{7}{1000} + \$\frac{8}{10000}$? Express the result in its lowest terms.

22. A butcher paid $\$1\frac{1}{2}$ for a sheep, $\$\frac{2}{4}$ for a calf, and $\$\frac{10}{8}$ for a lamb. What was the whole cost?

23. A owns $\frac{3}{7}$ of a vessel, B owns $\frac{2}{8}$ of it, and C owns $\frac{18}{56}$ of it. How much do the three own?

24. I sold $\frac{2}{5}$ of an acre to one man, $\frac{7}{15}$ of an acre to another, and $\frac{17}{16}$ of an acre to a third. How much did I sell to all?

257. *Problem.* — If a man spent $\$7\frac{3}{8}$ on Monday, $\$\frac{7}{10}$ on Tuesday, and $\$8\frac{8}{25}$ on Wednesday, how much did he spend in all?

SOLUTION.

$\$7\frac{3}{8} = \$7\frac{75}{200}$

$\$\frac{7}{10} = \$\frac{140}{200}$

$\$8\frac{8}{25} = \$8\frac{64}{200}$

$\$16\frac{79}{200}$

EXPLANATION. — Since the man spent $\$7\frac{3}{8}$ on Monday, $\$\frac{7}{10}$ on Tuesday, and $\$8\frac{8}{25}$ on Wednesday, he spent in all the sum of $\$7\frac{3}{8}$, $\$\frac{7}{10}$, and $\$8\frac{8}{25}$.

Since the fractions are dissimilar, etc. (256).

The least common multiple of the denominators is 200.

$\frac{3}{8}$ changed to 200ths $= \frac{75}{200}$, etc. (See 256, for work and arrangement.)

Now the sum of the fractions $= \frac{75+140+64}{200} = \frac{279}{200} = 1\frac{79}{200}$; and $\$1\frac{79}{200}$ added to \$15, the sum of the integers, equals $\$16\frac{79}{200}$, the sum spent.

258. Rule for Addition of Fractions.

I. *Change the given fractions to similar fractions having the least common denominator.*

II. *Add together the numerators, and under their sum write the least common denominator. The resulting fraction will be the sum required.*

III. *If any of the parts to be added are mixed numbers or integers, add the fractions and the integers separately, and unite the results for the entire sum required.*

Note 1. — If the integers or the mixed numbers to be added are small, they may be changed to improper fractions, and then added.

2. If the result of *any* operation in fractions contains an improper fraction, it must be changed to a mixed number; and every fraction in any result must be expressed in its lowest terms.

Problems.

Find the sum of—

25. $1\frac{1}{2}$, 2, $3\frac{4}{5}$.
26. $\frac{3}{5}$, $4\frac{2}{3}$, $6\frac{2}{5}$.
27. $5\frac{5}{8}$, $\frac{7}{8}$, $7\frac{8}{9}$.
28. $\frac{8}{11}$, $7\frac{7}{10}$, $8\frac{10}{11}$.
29. $9\frac{11}{12}$, $\frac{14}{15}$, $10\frac{1}{8}$.
30. $11\frac{9}{20}$, $\frac{18}{25}$, $15\frac{9}{40}$.
31. $16\frac{3}{28}$, $\frac{9}{7}$, 23.
32. 19, $\frac{5}{66}$, $33\frac{1}{99}$.
33. $44\frac{1}{10}$, 10, $100\frac{11}{100}$.

34. Find the amount of $13\frac{1}{20}$, $\frac{7}{15}$, 100, and $11\frac{1}{50}$.

35. What is the sum of $\$25\frac{1}{5}$, $\$\frac{3}{25}$, $\$11\frac{9}{30}$, $\$125\frac{3}{20}$.

36. Add together $10\frac{3}{16}$ pounds, $21\frac{9}{20}$ pounds, and 43 pounds.

37. How many tons are 25 tons, $45\frac{9}{20}$ tons, and $100\frac{5}{200}$ tons?

38. How many acres are in a farm which consists of $82\frac{7}{16}$ acres of cleared land, and $10\frac{5}{20}$ acres of woodland?

39. A grocer bought some flour for $\$18\frac{3}{4}$, sugar for $\$37\frac{3}{25}$, and tea for $\$12\frac{2}{3}$. What was the amount of his bill?

40. Richard walked $27\frac{3}{8}$ miles one day, 24 miles the next, and $30\frac{3}{80}$ miles the next. How far did he go?

41. How many yards are in three pieces of carpet that contain $52\frac{7}{12}$ yards, $69\frac{5}{36}$ yards, and $80\frac{2}{3}$ yards?

42. John has $\$7\frac{3}{4}$, Thomas has $\$3\frac{2}{5}$ more than John, and Henry has $\$9\frac{5}{8}$ more than both the others. How much has each, and how much have all of them?

SECTION XI.

SUBTRACTION OF FRACTIONS.

1. What is the difference between 7 yards and 5 yards? Between 7 eighths and 5 eighths?

2. Frank earned 7 eighths of a dollar, and spent 5 eighths of a dollar. How many eighths had he left?

3. How many ninths are 8 ninths less 3 ninths? $\frac{8}{9}$ less $\frac{3}{9}$? $\frac{7}{9} - \frac{3}{9}$? $\frac{7}{9} - \frac{4}{9}$?

4. What is the remainder of $\frac{4}{5} - \frac{2}{5}$? $\frac{5}{8} - \frac{3}{8}$? $\frac{6}{7} - \frac{2}{7}$?

5. What is the numerator of the remainder of $\frac{13}{12} - \frac{8}{12}$? What is the denominator?

6. How do you subtract one fraction from another fraction having the same denominator?

7. What is the unit of 3 fourths? Of 4 fifths? Can 3 fourths be taken directly from 4 fifths? Why?

8. To what similar fractions can $\frac{3}{4}$ and $\frac{4}{5}$ be changed? Change them to similar fractions, and find their difference.

9. To what similar fractions must $\frac{5}{6}$ and $\frac{2}{7}$ be changed before their difference can be found?

10. Change $\frac{7}{8}$ and $\frac{5}{8}$ to least similar fractions, and find their difference; $\frac{8}{9}$ and $\frac{5}{6}$; $\frac{9}{10}$ and $\frac{7}{8}$; $\frac{11}{12}$ and $\frac{8}{9}$.

11. A dealer bought a bushel of potatoes for $\$\frac{7}{8}$, and sold them for $\$\frac{9}{10}$. How much did he gain?

259. Analysis. — Since the dealer bought a bushel of potatoes for $\$\frac{7}{8}$, and sold them for $\$\frac{9}{10}$, he gained the difference between $\$\frac{9}{10}$ and $\$\frac{7}{8}$.

Since only similar fractions can be taken one from another, $\frac{9}{10}$ and $\frac{7}{8}$ must be changed to least similar fractions. $\frac{9}{10}$ changed to 40ths equals $\frac{36}{40}$; and $\frac{7}{8}$ changed to 40ths equals $\frac{35}{40}$. Now $\$\frac{36}{40}$ less $\$\frac{35}{40}$ equal $\$\frac{1}{40}$. Hence, he gained $\$\frac{1}{40}$.

12. If a grocer draws $\frac{4}{5}$ of a gallon of oil from a can containing $\frac{9}{10}$ of a gallon, how much is left?

13. How do you subtract dissimilar fractions, or fractions having different denominators?

260. Principles.

I. *Only fractional units of the same kind of things can be taken one from another.*

II. *Only similar fractions can be taken one from another.*

Written Exercises.

261. *Example.* — Find the difference between $\frac{11}{12}$ and $\frac{8}{9}$.

SOLUTION.

3)12, 9 **Denominators.**

4, 3

$3 \times 4 \times 3 = 36$ **L. c. d.**

$36 \div 12 = 3$, and $\frac{11}{12} \times \frac{3}{3} = \frac{33}{36}$

$36 \div 9 = 4$, and $\frac{8}{9} \times \frac{4}{4} = \frac{32}{36}$

$\frac{11}{12} - \frac{8}{9} = \frac{33}{36} - \frac{32}{36} = \frac{1}{36}$

EXPLANATION.—Since the fractions are dissimilar, they must be changed to similar fractions having the least common denominator.

The least common multiple of the denominators is 36.

$\frac{11}{12}$ changed to 36ths $= \frac{33}{36}$, and $\frac{8}{9}$ changed to 36ths $= \frac{32}{36}$.

Now the difference of the similar fractions is the difference in the number of 36ths expressed by the fractions, and is found by taking the less numerator from the greater; and $\frac{33-32}{36} = \frac{1}{36}$, the difference required.

Problems.

Find the difference between —

1. $\frac{4}{5}$ and $\frac{3}{4}$.
2. $\frac{5}{6}$ and $\frac{3}{5}$.
3. $\frac{8}{9}$ and $\frac{7}{8}$.
4. $\frac{9}{11}$ and $\frac{9}{10}$.
5. $\frac{11}{12}$ and $\frac{10}{11}$.
6. $\frac{14}{15}$ and $\frac{11}{12}$.
7. $\frac{11}{18}$ and $\frac{13}{15}$.
8. $\frac{17}{20}$ and $\frac{17}{24}$.
9. $\frac{23}{30}$ and $\frac{19}{25}$.
10. $\frac{19}{34}$ and $\frac{29}{51}$.
11. $\frac{26}{39}$ and $\frac{52}{78}$.
12. $\frac{49}{57}$ and $\frac{69}{95}$.

13. Subtract $\frac{14}{33}$ from $\frac{67}{121}$; $\frac{28}{63}$ from $\frac{91}{108}$; $\frac{37}{64}$ from $\frac{101}{112}$.

14. From $\frac{60}{143}$ take $\frac{19}{66}$; and from $\frac{100}{133}$ take $\frac{101}{171}$.

15. What is the remainder of $\frac{201}{242}$ less $\frac{201}{264}$? $\frac{261}{275}$ less $\frac{263}{330}$?

16. $\frac{91}{360}$ of a year — $\frac{15}{84}$ of a year leaves what part of a year?

17. What is $\$\frac{87}{100} - \$\frac{21}{50}$? $\frac{40}{160}$ of an acre — $\frac{3}{40}$ of an acre?

18. From a bottle containing $\frac{2}{3}$ of a quart of wine, there was taken $\frac{3}{8}$ of a quart. How much remained in it?

19. A boy gathered $\frac{25}{32}$ of a bushel of chestnuts, and sold $\frac{5}{8}$ of a bushel. What part of a bushel remained?

20. A merchant owned $\frac{8}{9}$ of a vessel, and sold $\frac{3}{5}$ of it. What part did he then own?

262. *Problem.* — From a field of $8\frac{3}{16}$ acres were sold $2\frac{3}{10}$ acres. How many acres remained?

SOLUTION.

$8\frac{3}{16} = 8\frac{15}{80}$ *acres* $= 7\frac{95}{80}$ *acres.*

$2\frac{3}{10} = 2\frac{24}{80}$ *acres* $= 2\frac{24}{80}$ *acres.*

$5\frac{71}{80}$ *acres.*

2)16, 10

8, 5

$2 \times 8 \times 5 = 80$, ***L. c. d., etc.***

EXPLANATION. — Since there were in the field $8\frac{3}{16}$ acres, and there were sold $2\frac{3}{10}$ acres, there then remained the difference between $8\frac{7}{16}$ acres and $2\frac{3}{10}$ acres.

Since the fractions are dissimilar, etc. (261).

The least common denominator is 80. $\frac{3}{16}$ changed to 80ths $= \frac{15}{80}$, and $\frac{3}{10} = \frac{24}{80}$.

Since $\frac{24}{80}$ cannot be taken from $\frac{15}{80}$, take 1 or $\frac{80}{80}$ from 8 ones in the minuend, leaving 7 ones; $\frac{80}{80} + \frac{15}{80} = \frac{95}{80}$, and $\frac{95}{80} - \frac{24}{80} = \frac{71}{80}$, which added to 5, the difference of the integers, equals $5\frac{71}{80}$ acres, the remainder required.

263. Rule for Subtraction of Fractions.

I. *Change the given fractions to similar fractions having the least common denominator.*

II. *Subtract the less numerator from the greater, and under the difference write the least common denominator. The resulting fraction will be the difference required.*

III. *If either term is a mixed number, subtract the fractions and the integers separately, and unite the results for the entire difference required.*

Problems.

Find the remainder of—

21. $1\frac{6}{7} - \frac{3}{4}$.
22. $2\frac{8}{9} - 1\frac{5}{6}$.
23. $5\frac{9}{10} - 3\frac{7}{8}$.
24. $9\frac{4}{15} - 5\frac{7}{12}$.
25. $11\frac{3}{5}$ less $8\frac{3}{5}$.
26. $17\frac{11}{30}$ less $11\frac{1}{20}$.
27. $25\frac{3}{36}$ less $17\frac{23}{24}$.
28. $33\frac{5}{42}$ less $23\frac{5}{63}$.
29. $100 - \frac{1}{1000}$.
30. $200 - \frac{199}{200}$.
31. $300\frac{1}{10} - \frac{99}{1000}$.
32. $400 - 399\frac{3}{50}$.
33. Subtract $40\frac{3}{82}$ from $41\frac{1}{123}$. From $100\frac{1}{10}$ take $99\frac{9}{10}$.
34. What number added to $53\frac{7}{11}$ equals $101\frac{2}{33}$?
35. How much less is $62\frac{4}{51}$ than $75\frac{5}{68}$? $71\frac{3}{38}$ than $80\frac{1}{57}$?
36. If $85\frac{1}{35}$ be taken from 100, what will be left? $10\frac{1}{10}$ from $11\frac{1}{11}$? $200\frac{1}{25}$ from 201? $88\frac{9}{14}$ from $110\frac{1}{7}$?

37. The minuend is $125\frac{3}{26}$, and the subtrahend is $111\frac{2}{39}$. Find the remainder and subtract it from the minuend.

38. The greater of two numbers is $137\frac{1}{5}\frac{1}{7}$, and the less is $136\frac{7}{78}$. What is the difference? Take it from the greater.

39. The sum of two numbers is $237\frac{5}{14}\frac{1}{1}$, and one of them is $137\frac{3}{94}$. What is the other? If one is $57\frac{3}{188}$?

40. I bought a ton of coal for $\$7\frac{3}{10}$, and a cord of wood for $\$5\frac{7}{8}$. What was the difference in price?

41. From a cask of wine containing $60\frac{1}{16}$ gallons $20\frac{3}{4}$ gallons were drawn. How much remained?

42. On Monday a ship sailed $75\frac{7}{80}$ miles, and on Tuesday as far, less $3\frac{11}{180}$ miles. How far did it sail on Tuesday?

43. If a farmer should sell $44\frac{3}{16}$ acres from a farm containing 160 acres, how many acres would remain?

44. An agent sold an estate for $\$7520\frac{75}{100}$, for which he had paid \$8000. How much did he lose?

45. A city property was sold for $\$12050\frac{7}{8}$, which was at a gain of $\$505\frac{25}{100}$. What was paid for it?

EXERCISES IN ADDITION AND SUBTRACTION OF FRACTIONS.

Written Exercises.

264. *Problem.* — A farmer owned $76\frac{1}{16}$ acres, then sold $20\frac{5}{6}$ acres, and bought $55\frac{31}{160}$ acres. What had he then?

SOLUTION.

$76\frac{1}{16} = 76\frac{3}{48}$ *acres owned.*

$20\frac{5}{6} = 20\frac{40}{48}$ *acres sold.*

$55\frac{11}{48}$ *acres remaining.*

EXPLANATION.—Since the farmer owned $76\frac{1}{16}$ acres, and then sold $20\frac{5}{6}$ acres, he had remaining the difference between $76\frac{1}{16}$ and $20\frac{5}{6}$ acres, which is $55\frac{11}{48}$ (262).

$55\frac{11}{48} = 55\frac{110}{480}$ *acres.*

$55\frac{31}{160} = 55\frac{93}{480}$ *acres bought.*

$110\frac{203}{480}$ *acres in all.*

If he then bought $55\frac{31}{160}$ acres, he then had the sum of $55\frac{11}{48}$ acres and $55\frac{31}{160}$ acres, which is $110\frac{203}{480}$ acres, the number of acres he then had (257, for work).

Problems.

Find the result of—

1. $\frac{7}{12} + \frac{11}{15} + \frac{21}{20} - \frac{59}{60}$.
2. $\frac{13}{17} + \frac{11}{34} - \frac{18}{51} + \frac{2}{3}$.
3. $\frac{56}{57} - (\frac{8}{19} + \frac{9}{38}) - \frac{11}{76}$.
4. $\frac{11}{65} + \frac{3}{13} - \frac{7}{39} + \frac{5}{26}$.
5. $7\frac{3}{7} - (1\frac{3}{5} + 2\frac{4}{7} + 2\frac{3}{8})$.
6. $28\frac{3}{10} + 5\frac{7}{15} - (35 - 17\frac{9}{20})$.
7. $78\frac{7}{40} - (60\frac{1}{20} + 15\frac{7}{8} - 4\frac{3}{5})$.
8. $(95\frac{2}{15} - 25\frac{7}{20}) + (10\frac{5}{6} - 9\frac{8}{9})$.

9. To what must $\frac{6}{7}$ be added that the sum may be $\frac{7}{8}$?
10. What number added to $17\frac{1}{9} + 3\frac{5}{6}$ makes $25\frac{7}{15}$?
11. From the sum of $16\frac{5}{9} + 5\frac{7}{12}$ take $20 - 17\frac{17}{18}$.
12. To the remainder of $20\frac{2}{3} - \frac{16}{17}$ add $18 + \frac{59}{51}$.
13. From the sum of $11\frac{3}{10}$ and $9\frac{9}{20}$ take their difference.
14. To the difference of $15\frac{5}{6}$ and $12\frac{5}{12}$ add their sum.
15. The sum of two numbers is $55\frac{7}{8}$ feet, and the greater number is $44\frac{7}{12}$ feet. What is the less? Take the less from the greater. Add their sum to their difference.
16. The difference between two fractions is $\frac{4}{15}$ of a foot, and the greater fraction is $\frac{9}{11}$ of a foot. What is the less?
17. The sum of three numbers is $275\frac{11}{24}$ tons; one of them is $135\frac{11}{18}$ tons, and another is $125\frac{11}{12}$. What is the third?
18. From 200 pounds take the difference between $196\frac{1}{16}$ pounds and $28\frac{7}{16}$ pounds. Find the sum of the three numbers.
19. A mason had \$100, paid $\$25\frac{3}{4}$ for a month's rent, and earned enough to make \$125. What did he earn?
20. From two pieces of muslin, each containing $33\frac{3}{4}$ yards, a merchant sold $50\frac{2}{3}$ yards. How many yards remained?

21. A man had in bank $350\frac{25}{100}$, then earned enough to pay $500 on his house, lacking $25\frac{7}{8}$. What did he earn?

22. James started on a journey with $50; he paid $25 in car fare, and $15\frac{7}{10}$ in hotel bills. What had he left?

23. A mechanic saved $30 from a month's wages, and then bought a coat for $12\frac{75}{100}$, and a pair of boots for $6\frac{3}{4}$. How much remained?

24. A man had a journey of 150 miles to make. After going $89\frac{3}{8}$ miles by railroad, and $17\frac{9}{10}$ miles by stage, how far had he still to travel?

SECTION XII.

MULTIPLICATION OF FRACTIONS.

CASE I.

To Multiply a Fraction by an Integer.

1. How many fourths of a pear are 3 times 1 fourth?

2. What part of a pear are 2 times 2 fifths of a pear?

3. If you should give $\frac{1}{4}$ of a cake to each of 9 boys, how many fourths of a cake would be needed? How many cakes?

4. At $\$\frac{3}{10}$ a pound, what do 5 pounds of coffee cost?

265. Analysis. — Since 1 pound of coffee costs $\$\frac{3}{10}$, 5 pounds cost 5 times $\$\frac{3}{10}$, which are $\$\frac{15}{10}$, or $\$1\frac{1}{2}$. Hence, etc.

5. How many fifths are 3 times $\frac{2}{5}$? 5 times $\frac{3}{5}$?

6. How many are 5 times $\frac{3}{7}$? 6 times $\frac{5}{8}$? 7 times $\frac{3}{10}$?

7. At $\$\frac{3}{4}$ a bushel, what will 4 bushels of apples cost? 6 bushels? 8 bushels? 10 bushels?

8. What is the cost of 9 oranges at $5\frac{1}{2}$ cents each?

266. Analysis. — Since 1 orange costs $5\frac{1}{2}$ cents, 9 oranges cost 9 times $5\frac{1}{2}$ cents.

9 times $\frac{1}{2}$ cent are $\frac{9}{2}$ of a cent, or $4\frac{1}{2}$ cents; 9 times 5 cents are 45 cents; and 45 cents plus $4\frac{1}{2}$ cents are $49\frac{1}{2}$ cents. Hence, etc.

9. What is the result, if the numerator of $1\frac{3}{12}$ is multiplied by 3? Express it in its lowest terms.

10. What is the result, if the denominator of $\frac{3}{12}$ is divided by 3? In what two ways, then, may a fraction be multiplied by an integer?

267. Principle.

A fraction is multiplied by multiplying its numerator or dividing its denominator by any integer.

Written Exercises.

268. *Example* 1.—Find the product of $1\frac{3}{14}$ multiplied by 7.

SOLUTION.

$$\frac{13}{14} \times 7 = \frac{13 \times \cancel{7}}{\cancel{14}_{2}} = \frac{13}{2} = 6\frac{1}{2}.$$

EXPLANATION. — 7 times $1\frac{3}{14} = \frac{13 \times 7}{14}$, which by cancellation, or by dividing the denominator by the integer $= \frac{13}{2}$, or $6\frac{1}{2}$.

269. *Example* 2. — Multiply $13\frac{9}{11}$ by 8.

SOLUTION.

$$\begin{array}{r} 13\frac{9}{11} \\ \times \quad 8 \\ \hline 6\frac{6}{11} \\ 104 \\ \hline 110\frac{6}{11}. \end{array}$$

$\frac{9}{11} \times 8 = \frac{72}{11} = 6\frac{6}{11}$.

EXPLANATION. — Since $13\frac{9}{11}$ equals $13 + \frac{9}{11}$, the product of $13\frac{9}{11}$ by 8 is the same as 8 times $\frac{9}{11}$ plus 8 times 13.

8 times $\frac{9}{11} = 6\frac{6}{11}$; and 8 times 13 are 104. $6\frac{6}{11}$ added to 104 equals $110\frac{6}{11}$, the product required.

270. Rule to Multiply a Fraction by an Integer.

I. *Multiply the numerator of the fraction, or divide the denominator, by the integer.*

II. *If the multiplicand is a mixed number, multiply the integral part of the multiplicand and the fractional part separately, and unite the results for the entire product required.*

Note 1.—The operation may be shortened by cancellation.

2. If the mixed number is small, it may be changed to an improper fraction, and then multiplied.

Problems.

Multiply —

1. $\frac{5}{6}$ by 4.	4. $1\frac{1}{12}$ by 10.	7. $17\frac{2}{3}$ by 6.	10. $112\frac{7}{15}$ by 25.
2. $\frac{7}{8}$ by 6.	5. $1\frac{3}{18}$ by 12.	8. $19\frac{8}{9}$ by 8.	11. $234\frac{8}{25}$ by 75.
3. $\frac{8}{9}$ by 8.	6. $\frac{27}{24}$ by 20.	9. $25\frac{9}{10}$ by 9.	12. $350\frac{22}{57}$ by 95.

13. Multiply $\frac{3}{44}$ by 4, by 6, by 8, by 10, by 12, and by 22.

14. Find the product of $3\frac{2}{11} \times 22$; $8\frac{3}{35} \times 42$; $20\frac{3}{20} \times 40$.

15. How much is 7 times $\frac{3}{11}$? 8 times $\frac{5}{15}$? 9 times $18\frac{11}{81}$?

16. Find the product of $\frac{12}{14}\frac{1}{3} \times 33$; $\frac{57}{100} \times 20$; $10\frac{11}{200} \times 400$; $200\frac{23}{300} \times 90$; $350\frac{19}{40} \times 800$; $500\frac{21}{500} \times 1000$.

17. 1 rod equals $5\frac{1}{2}$ yards. How many yards in 15 rods?

18. At $\$\frac{7}{10}$ a bushel, what do 8 bushels of corn cost?

19. At the rate of $6\frac{3}{4}$ miles an hour, how far could you drive in 5 hours? In 6 hours? In 7 hours?

20. If a flour-dealer pays $\$7\frac{3}{10}$ a barrel for XX flour, what do 50 barrels cost? 200 barrels? 500 barrels?

CASE II.

To Multiply an Integer by a Fraction.

1. What is $\frac{1}{2}$ of 4 cents? $\frac{1}{3}$ of 9 yards? $\frac{1}{5}$ of \$10?

2. $\frac{1}{3}$ of 2 inches is what part of 1 inch?

271. Analysis. — $\frac{1}{3}$ of 2 inches is 2 times $\frac{1}{3}$ of an inch, which are $\frac{2}{3}$ of an inch. Hence, etc.

3. How much is 1 time 5 miles? $\frac{1}{2}$ of 1 time 5 miles? $\frac{1}{3}$ of 5 miles? $\frac{1}{4}$ of 5 miles? $\frac{1}{5}$ of 9 miles?

4. Multiplying by $\frac{1}{8}$ gives the same result as dividing by what number? By $\frac{1}{4}$? By $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{9}$? $\frac{1}{12}$?

5. At \$6 a ton for coal, what is the cost of $\frac{3}{4}$ of a ton?

272. Analysis. — Since 1 ton costs \$6, $\frac{3}{4}$ of a ton cost 3 times $\frac{1}{4}$ of \$6, or $\frac{3}{4}$ of \$6. $\frac{1}{4}$ of \$6 is $\$\frac{6}{4}$, and $\frac{3}{4}$ of \$6 are 3 times $\$\frac{6}{4}$, which are $\$1\frac{8}{4}$, or $\$4\frac{2}{4}$, or $\$4\frac{1}{2}$. Hence, etc.

6. If an acre yields 30 bushels of wheat, what will $\frac{2}{3}$ of an acre yield? $\frac{3}{4}$? $\frac{4}{5}$? $\frac{5}{6}$? $\frac{6}{7}$? $\frac{7}{8}$? $\frac{9}{10}$? $\frac{11}{12}$? $\frac{13}{15}$?

7. If a man can do a piece of work in 40 days, in what time can he do $\frac{2}{3}$ of it? $\frac{3}{4}$ of it? $\frac{5}{9}$? $\frac{11}{12}$? $\frac{13}{21}$?

8. Multiplying \$20 by $\frac{3}{4}$ is the same as taking what part of \$20? Multiplying by $\frac{4}{5}$? By $\frac{9}{10}$? $\frac{13}{15}$?

9. At 12 cents a pound, what is the cost of $4\frac{3}{8}$ pounds of sugar?

273. ***Analysis.*** — Since one pound costs 12 cents, $4\frac{3}{8}$ pounds cost $4\frac{3}{8}$ times 12 cents. 4 times 12 cents are 48 cents; and $\frac{3}{8}$ of 12 cents are 3 times $\frac{1}{8}$ of 12 cents, or 3 times $\frac{1}{8}$ cents, which are $\frac{3}{8}$ cents, or $4\frac{1}{2}$ cents; and $4\frac{1}{2}$ cents added to 48 cents equal $52\frac{1}{2}$ cents.

10. If one acre yields 40 bushels of corn, how many bushels will $3\frac{1}{2}$ acres yield? $4\frac{2}{3}$ acres? $5\frac{3}{4}$? $6\frac{7}{8}$? $8\frac{9}{10}$?

274. Principle.

An integer is multiplied by a fraction by taking such part of the integer as the fraction indicates.

Written Exercises.

275. *Example* 1. — Multiply 36 by $\frac{3}{8}$, or find $\frac{3}{8}$ of 12.

SOLUTION.

$$36 \times \frac{3}{8} = \frac{^{9}\cancel{36} \times 3}{\cancel{8}_{2}} = \frac{27}{2} = 13\frac{1}{2}.$$

EXPLANATION. — $\frac{3}{8}$ of $36 = 36 \times \frac{3}{8}$. Since $\frac{3}{8}$ is $\frac{1}{8}$ of 3, $\frac{3}{8}$ of 36 equals $\frac{1}{8}$ of 3 times 36, or $\frac{36 \times 3}{8}$, which by cancellation equals $\frac{27}{2}$, or $13\frac{1}{2}$, the product required.

276. *Example* 2. — Find the product of 36 multiplied by $8\frac{3}{8}$.

SOLUTION.

$$\begin{array}{r} 36 \\ 8\frac{3}{8} \\ \hline 13\frac{1}{2} \\ 288 \\ \hline 301\frac{1}{2}. \end{array}$$

$$\frac{^{9}\cancel{36} \times 3}{\cancel{8}_{2}} = \frac{27}{2} = 13\frac{1}{2}.$$

EXPLANATION.—Since $8\frac{3}{8}$ equals $8 + \frac{3}{8}$, the product of 36 by $8\frac{3}{8}$ equals $\frac{3}{8}$ of 36 plus 8 times 36. $\frac{3}{8}$ of $36 = 13\frac{1}{2}$; and 8 times 36 $= 288$. $13\frac{1}{2}$ added to 288 equals $301\frac{1}{2}$, the product required.

277. Rule to Multiply an Integer by a Fraction.

I. *Multiply the integer by the numerator of the fraction, and divide the product by the denominator.*

II. *If the multiplier is a mixed number, multiply by the integral part of the multiplier and the fractional part separately, and unite the results for the entire product required.*

Problems.

Multiply —

1. 15 by $\frac{4}{5}$.
2. 20 by $\frac{5}{8}$.
3. 32 by $\frac{7}{8}$.
4. 36 by $\frac{9}{10}$.
5. 44 by $\frac{10}{11}$.
6. 52 by $\frac{13}{14}$.
7. 62 by $7\frac{5}{6}$.
8. 75 by $8\frac{7}{8}$.
9. 96 by $9\frac{8}{9}$.
10. 135 by $30\frac{1}{45}$.
11. 246 by $45\frac{11}{211}$.
12. 357 by $84\frac{13}{17}$.

13. Multiply 21 by $\frac{2}{3}$, by $\frac{3}{4}$, by $\frac{5}{6}$, by $\frac{6}{7}$, by $\frac{8}{9}$, by $\frac{9}{10}$.

14. Find the product of $20 \times \frac{3}{10}$; $30 \times 8\frac{3}{4}$; 45 by $10\frac{7}{15}$.

15. How much is $\frac{3}{4}$ of 87? $108 \times \frac{3}{10}$? $10\frac{1}{10}$ times 175?

16. At \$8 a cord, what will $\frac{11}{16}$ of a cord of wood cost?

17. If a man do a piece of work in 20 days, in what time can he do $\frac{2}{3}$ of it? $\frac{3}{4}$ of it? $\frac{7}{8}$? $\frac{8}{9}$? $\frac{9}{10}$? $\frac{11}{12}$? $\frac{14}{15}$?

18. At \$18 a ton for hay, what must be paid for $\frac{1}{2}$ of a ton? For 2 tons? For $2\frac{1}{2}$ tons? $10\frac{1}{3}$ tons? $20\frac{3}{20}$ tons?

19. If 6 men do a piece of work in $5\frac{2}{3}$ days, how many men will it take to do it in 1 day?

20. In a piece of muslin are 34 yards. How many yards are in $2\frac{1}{2}$ pieces? $5\frac{3}{4}$ pieces? $17\frac{3}{10}$ pieces?

CASE III.

To Multiply a Fraction by a Fraction.

1. If $\frac{1}{2}$ of an orange is cut into 2 equal pieces, what part of the whole orange is each piece?

2. If $\frac{1}{2}$ of a pear is cut into 3 equal pieces, what part of the whole pear is each piece? If into 4 equal pieces?

3. What part of 1 is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{4}$ of $\frac{1}{2}$?

4. Frank had $\frac{1}{4}$ of a cake, and gave $\frac{1}{2}$ of it to his sister. What part of the cake did his sister receive?

278. Analysis. — Since $\frac{1}{4}$ of a cake is one of 4 equal parts of the cake, if 1 fourth is divided into 2 equal parts, each of these parts is equal to one of 8 equal parts of the cake, or $\frac{1}{8}$ of the cake. Hence, etc.

5. How much is $\frac{1}{2}$ of 1 mile? $\frac{1}{2}$ of 1 ton? $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{5}$? $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{1}{3}$ of $\frac{1}{5}$? $\frac{1}{3}$ of $\frac{1}{8}$?

6. How much is $\frac{1}{2}$ of 2 tons? $\frac{1}{2}$ of 4 feet? $\frac{1}{2}$ of 2 thirds? $\frac{1}{2}$ of 4 fifths? $\frac{1}{2}$ of $\frac{6}{7}$? $\frac{1}{2}$ of $\frac{8}{9}$? $\frac{1}{3}$ of $\frac{6}{7}$? $\frac{1}{4}$ of $\frac{8}{9}$?

7. What part of 1 is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{2}$ of $\frac{2}{3}$ is how many times $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{2}$ of $\frac{2}{3}$ is what part of 1? $\frac{1}{2}$ of $\frac{4}{5}$?

8. If a pound of tea costs $\$\frac{4}{5}$, what is the cost of $\frac{2}{3}$ of a pound?

279. Analysis. — Since 1 pound of tea costs $\$\frac{4}{5}$, the cost of $\frac{2}{3}$ of a pound is $\frac{2}{3}$ of $\$\frac{4}{5}$.

$\frac{2}{3}$ of $\frac{4}{5}$ are 2 times $\frac{1}{3}$ of $\frac{4}{5}$. $\frac{1}{3}$ of $\frac{4}{5}$ is $\frac{4}{15}$; $\frac{1}{3}$ of $\frac{4}{5}$ is 4 times $\frac{1}{15}$, or $\frac{4}{15}$, and $\frac{2}{3}$ of $\frac{4}{5}$ are 2 times $\frac{4}{15}$, which are $\frac{8}{15}$. Hence, etc.

9. In the last problem, how may 8, the numerator of the result, be obtained from the terms of the two fractions? How may 15, the denominator, be obtained?

10. A captain owned $\frac{5}{7}$ of a schooner, and sold $\frac{3}{4}$ of his share. What part of the schooner did he sell?

Definitions.

280. A ***Simple Fraction*** is a fraction whose numerator and denominator are both integers.

Thus, $\frac{2}{3}$, $\frac{3}{2}$, $\frac{4}{7}$, $\frac{24}{7}$ are *simple fractions.*

281. A ***Compound Fraction*** is a fraction of a fraction.

A compound fraction consists of two or more fractions connected by *of.*

Thus, $\frac{2}{3}$ *of* $\frac{4}{5}$, $\frac{5}{6}$ *of* $\frac{6}{8}$, $\frac{8}{9}$ *of* $2\frac{1}{2}$ are *compound fractions.*

282. Principle.

A fraction is multiplied by a fraction by taking such part of the fraction as the fractional multiplier indicates.

Written Exercises.

283. *Example* 1. — Find the value of $\frac{4}{5}$ of $\frac{5}{6}$.

SOLUTION.

$\frac{4}{5}$ of $\frac{5}{6} = \frac{\cancel{5}}{\cancel{6}_{3}} \times \frac{\cancel{4}^{2}}{\cancel{5}} = \frac{2}{3}$.

EXPLANATION. — $\frac{1}{5}$ of $\frac{5}{6}$ equals 4 times $\frac{1}{5}$ of $\frac{5}{6}$; and since $\frac{1}{5}$ of $\frac{5}{6}$ is $\frac{5}{6 \times 5}$, $\frac{4}{5}$ of $\frac{5}{6}$ is $\frac{5 \times 4}{6 \times 5}$, which by cancellation equals $\frac{2}{3}$, the value required.

284. *Example* 2. — Find the product of $\frac{7}{15}$, $4\frac{1}{2}$, and 8.

SOLUTION.

$4\frac{1}{2} = \frac{9}{2}$; $8 = \frac{8}{1}$.

$\frac{7}{\cancel{15}_{5}} \times \frac{\cancel{9}^{3}}{\cancel{2}} \times \frac{\cancel{8}^{4}}{1} = \frac{84}{5} = 16\frac{4}{5}$.

EXPLANATION.—The mixed number $4\frac{1}{2}$ equals $\frac{9}{2}$, and the integer 8, changed to the form of a fraction, equals $\frac{8}{1}$.

By cancelling and multiplying, the product is $\frac{84}{5}$, which equals $16\frac{4}{5}$, the result required.

285. General Rule for Multiplication of Fractions.

I. *Change integers and mixed numbers to improper fractions, and compound fractions to simple fractions.*

II. *Cancel the factors common to the numerators and denominators. Then multiply the numerators together for a new numerator, and the denominators together for a new denominator. The resulting fraction will be the product required.*

Note. — The problems under Case I and Case II may be solved according to the general rule by changing the integers to fractions.

Problems.

Find the value of—

1. $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{1}{5}$.
2. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.
3. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{5}{6}$.
4. $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$.
5. $\frac{4}{5}$ of $\frac{6}{7}$ of $\frac{8}{9}$.
6. $\frac{5}{6}$ of $\frac{9}{10}$ of $1\frac{1}{5}$.
7. $\frac{11}{21}$ of $\frac{28}{33}$ of $\frac{45}{64}$.
8. $\frac{17}{30}$ of $\frac{40}{51}$ of $\frac{63}{70}$.
9. $\frac{25}{36}$ of $\frac{54}{55}$ of $\frac{77}{91}$.

10. Change the compound fraction $\frac{1}{2}\frac{7}{7}$ of $\frac{8}{8}\frac{1}{5}$ to a simple fraction; $\frac{1}{2}\frac{9}{5}$ of $\frac{7}{1}\frac{5}{7}\frac{}{1}$; $\frac{7}{13}$ of $\frac{3}{5}\frac{9}{6}$ of $5\frac{3}{5}$; $\frac{9}{11}$ of $23\frac{5}{6}$ times $\frac{1}{2}\frac{}{2}$.

11. At $\$\frac{7}{10}$ a yard, what is the cost of $\frac{5}{8}$ of a yard of cloth?

12. If a bushel of apples is worth $\$\frac{9}{10}$, what is the value of $\frac{3}{5}$ of 4 bushels? $\frac{3}{4}$ of $\frac{5}{6}$ of a bushel?

Find the product of—

13. $\frac{4}{5} \times 6$; $\frac{5}{4} \times \frac{3}{10}$.

14. $2\frac{4}{5} \times \frac{6}{7}$; $4\frac{1}{2} \times \frac{8}{9}$.

15. $\frac{4}{5}$ of $6 \times \frac{5}{6}$ of $\frac{9}{10}$.

16. $\frac{6}{7}$ of $2\frac{4}{5} \times \frac{8}{9}$ of $4\frac{1}{2}$.

17. $\frac{2}{3}$ of $4\frac{1}{2}$ of $\frac{6}{7} \times 6\frac{1}{2} \times \frac{8}{13}$.

18. $\frac{3}{5}$ of $2\frac{2}{9} \times 6\frac{2}{3}$ times $\frac{9}{10}$.

19. $12\frac{3}{5} \times \frac{3}{7}$ of $4\frac{1}{5} \times 1\frac{1}{9}$.

20. $3\frac{1}{3} \times 9 \times 2\frac{2}{5}$ times $11\frac{2}{3}$.

21. Multiply together $4\frac{1}{2}$, $\frac{2}{3}$ of $\frac{4}{7}$, and $18\frac{3}{5}$.

22. Find the continued product of $4\frac{2}{3}$, $1\frac{1}{7}$, $\frac{2}{8}$, and $3\frac{1}{5}$ times 10; of 22, $7\frac{2}{3}$, $\frac{4}{4}\frac{5}{6}$, and $\frac{9}{1}\frac{}{1}$ of 15.

23. If the multiplicand is 5 times $\$7\frac{1}{5}$, and the multiplier is $\frac{3}{4}$ of $5\frac{1}{3}$ times $\frac{7}{8}$, what is the product?

24. The factors are $\frac{7}{8}$ of $2\frac{2}{11}$, $5\frac{1}{4}$ times $3\frac{2}{3}$, and $7\frac{2}{7}$ times $1\frac{1}{7} \times \frac{1}{16}$. What is the product?

25. What is the cost of $5\frac{1}{4}$ pounds of flour at $5\frac{1}{4}$ cents a pound? Of $8\frac{2}{3}$ yards of calico at $12\frac{1}{2}$ cents a yard?

26. Find the cost of 20 barrels of apples at $\$2\frac{2}{3}$ a barrel; at $\$3\frac{1}{4}$ a barrel; at $\$3\frac{3}{8}$; at $\$4\frac{3}{16}$; at $\$5\frac{25}{100}$; at $\$6\frac{7}{20}$.

27. What is the cost of $\frac{3}{4}$ of 9 pounds of honey at $37\frac{1}{2}$ cents a pound? $\frac{2}{7}$ of $2\frac{1}{10}$ pounds at $2\frac{1}{2}$ times $18\frac{3}{4}$ cents?

28. How many yards of muslin are in $18\frac{2}{3}$ pieces, each containing $34\frac{3}{4}$ yards? In $24\frac{3}{4}$ pieces? In $37\frac{3}{10}$ pieces?

29. If you buy a watch for $\$75\frac{3}{4}$, and sell it for $\frac{3}{4}$ of its cost, what do you get for it? If for $\frac{6}{5}$ of its cost?

30. If $2\frac{5}{8}$ tons of hay are cut from an acre of ground, how much hay will $10\frac{2}{7}$ acres yield? $7\frac{5}{7}$ times $3\frac{1}{9}$ acres? $5\frac{1}{7}$ times $2\frac{1}{12}$ acres $\times \frac{12}{25}$? $\frac{9}{10}$ of $17\frac{1}{7}$ acres $\times \frac{35}{54}$?

EXERCISES IN ADDITION, SUBTRACTION, AND MULTIPLICATION OF FRACTIONS.

Written Exercises.

286. *Problem.* — From a farm containing $50\frac{3}{8}$ acres were sold $30\frac{7}{10}$. What was the value of the remainder at $\$75\frac{3}{4}$ per acre?

SOLUTION.

$50\frac{3}{8} = 50\frac{15}{40} = 49\frac{55}{40}$ *acres.*

$30\frac{7}{10} = 30\frac{28}{40} = 30\frac{28}{40}$ *acres.*

$19\frac{27}{40}$ *acres.*

$\$75\frac{3}{4} \times 19\frac{27}{40} = \$\frac{303}{4} \times \frac{787}{40} = \$\frac{238461}{160} = \$1490\frac{61}{160}$.

EXPLANATION.—Since from $50\frac{3}{8}$ there are sold $30\frac{7}{10}$ acres, there remained the difference between $50\frac{3}{8}$ acres, or $49\frac{55}{40}$, and $30\frac{7}{10}$, or $30\frac{28}{40}$, which is $19\frac{27}{40}$ acres (262).

Since the selling price of one acre was $\$75\frac{3}{4}$, the selling price of $19\frac{27}{40}$ acres was $19\frac{27}{40}$ times $\$75\frac{3}{4}$, which are $\$1490\frac{61}{160}$, the selling price required.

Problems.

Find the result of —

1. $2\frac{3}{4} \times 5\frac{2}{3} + 1\frac{3}{8} - \frac{7}{12}$.
2. $(3\frac{5}{9} - 2\frac{8}{15}) \times 7\frac{1}{2} + 3\frac{5}{6}$.
3. $(12\frac{6}{7} + 8\frac{2}{3} \times 2\frac{1}{13}) - 25\frac{1}{3}$.
4. $75\frac{2}{5} - 2\frac{9}{10} + (8\frac{1}{6} \times 3\frac{3}{7})$.
5. $(8\frac{5}{8} \times 2\frac{10}{23}) - (\frac{6}{7}$ of $3\frac{2}{11})$.
6. $100\frac{1}{10} - (16\frac{7}{8} + 30\frac{3}{5} - 8\frac{5}{10})$.

7. What is the product of $5\frac{4}{7}$ times the sum of $3\frac{4}{19} + 6\frac{3}{4}$?

8. From $\frac{3}{17}$ of $6\frac{4}{5}$ take $20\frac{3}{7}$ less $19\frac{8}{35}$.

9. The sum of three fractions is 20; the least is $5\frac{3}{13}$, the greatest is $8\frac{7}{17}$. What is the product of the three?

10. William earned in a month $5\frac{3}{5}$ times $\$4\frac{2}{7}$ less $\$3\frac{75}{100}$. What amount did he earn? How much less than \$25?

11. Find the sum of $10\frac{15}{17} \times 7\frac{2}{7}$ and $7\frac{7}{50} - 6\frac{99}{100}$. The difference. Multiply their sum by their difference.

12. What number added to $\frac{3}{10}$ of $4\frac{2}{7}$ times $10\frac{1}{2}$ equals

20? Find the difference between the two numbers. Their product. The product of their sum and difference.

13. $\frac{3}{11}$ of $14\frac{2}{3}$ times $20\frac{3}{5}$ tons are how many more than $70\frac{3}{100}$ tons? How many less than $90\frac{19}{20}$ tons?

14. The sum of four fractions is $100\frac{3}{100}$. The first is $20\frac{7}{25}$, the second is $\frac{3}{7}$ of $39\frac{9}{10}$, and the third is $2\frac{7}{10}$ times $20\frac{5}{9}$. What is the fourth? What is their product?

15. How much less than $\$20\frac{7}{10}$ does a farmer receive for 6 barrels of apples at $\$2\frac{7}{8}$ a barrel?

16. If I buy $\frac{3}{8}$ of $10\frac{2}{3}$ yards at $\frac{3}{4}$ of 25 cents a yard, how much less than a dollar does it cost?

17. A merchant bought 5 pieces of muslin, each containing $34\frac{7}{8}$ yards, and sold $125\frac{7}{36}$ yards. How much remained?

18. A clerk's salary is $\$62\frac{50}{100}$ a month, and his expenses are $\$45\frac{7}{8}$. What can he save in 10 months?

19. A mechanic's wages were $\$3\frac{3}{8}$ a day, and he made $\$\frac{3}{10}$ more by over-work. What did he earn in a week?

20. How much must be paid for 12 tons of coal at $\$7\frac{3}{10}$ a ton, and 20 cords of wood at $\$5\frac{3}{8}$ a cord?

21. A man having $5000, bought a farm of 75 acres at $\$62\frac{3}{4}$ an acre. How much had he left?

22. From a bin containing $20\frac{1}{2}$ bushels of wheat there were sold $8\frac{1}{4}$ bushels at $\$1\frac{1}{3}$ a bushel, and the balance at $\$1\frac{1}{7}$ a bushel. What was the entire selling price?

23. If two men start from the same place, and go in the same direction, one $8\frac{3}{4}$ miles an hour, and the other $6\frac{2}{3}$ miles an hour, how far apart will they be in $5\frac{1}{2}$ hours?

24. If two men are 100 miles apart, and walk towards each other, one $3\frac{1}{8}$ miles an hour, and the other $2\frac{3}{4}$ miles, how far apart will they be in 16 hours?

25. In one farm are $25\frac{7}{8}$ acres; in a second 3 times as many, lacking $7\frac{11}{160}$ acres; in a third as many as are in both the other two. How many acres are in the three farms?

SECTION XIII.

DIVISION OF FRACTIONS.

CASE I.

To Divide a Fraction by an Integer.

1. If I divide $\frac{2}{3}$ of a cake between two boys, what part of $\frac{2}{3}$ does each boy get? What part of the cake?

2. How much is $\frac{1}{2}$ of $\frac{2}{3}$? $\frac{1}{2}$ of $\frac{2}{3}$ is what part of 1?

3. What part of $\frac{3}{5}$ is found by dividing it by 3? What is the quotient of $\frac{3}{5}$ divided by 3?

4. Dividing $\frac{5}{8}$ by 5 is taking what part of $\frac{5}{8}$? It is the same as multiplying by what fraction?

5. How much is $\frac{1}{6}$ of $\frac{12}{17}$, or $\frac{12}{17}$ divided by 6? $\frac{1}{7}$ of $\frac{21}{25}$, or $\frac{21}{25}$ divided by 7? $\frac{1}{8}$ of $\frac{24}{30}$? $\frac{1}{9}$ of $\frac{27}{35}$?

6. Dividing the numerator of a fraction has what effect upon the value of a fraction? Why?

7. If $\frac{1}{4}$ of a cake is cut into two equal parts, what part of $\frac{1}{4}$ is each piece? What part of the cake?

8. How much is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{3}$ divided by 2? $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{1}{4}$ divided by 3? $\frac{1}{4}$ of $\frac{1}{5}$? $\frac{1}{5}$ divided by 4?

9. If 3 brooms sell for $\$\frac{5}{8}$, what is the price of 1 broom?

287. Analysis. — Since 3 brooms sell for $\$\frac{5}{8}$, the price of 1 broom is $\frac{1}{3}$ of $\$\frac{5}{8}$, which is $\$\frac{5}{24}$. Hence, etc.

10. Multiplying the denominator of a fraction has what effect upon the value of the fraction? Why?

288. Principle.

A fraction is divided by dividing its numerator or multiplying its denominator by any integer.

Written Exercises.

289. *Example* 1. — Divide $\frac{14}{15}$ by 7.

SOLUTION.

$\frac{14}{15} \div 7 = \frac{14 \div 7}{15} = \frac{2}{15}$.

Or, $\frac{14}{15} \div 7 = \frac{\cancel{14}^{2}}{15 \times \cancel{7}_{1}} = \frac{2}{15}$.

EXPLANATION. — Since a fraction is divided by dividing the numerator, divide the numerator by 7, giving $\frac{2}{15}$, the quotient required. Or,

Since a fraction is divided by multiplying the denominator, $\frac{14}{15}$ divided by 7 equals $\frac{1}{7}$ of $\frac{14}{15}$, or $\frac{14}{15 \times 7}$, which by cancellation equals $\frac{2}{15}$.

290. *Example* 2. — What is the quotient of $166\frac{1}{4}$ divided by 5?

SOLUTION.

$166\frac{1}{4} = \frac{665}{4}$.

$\frac{665}{4} \div 5 = \frac{\cancel{665}^{133}}{4 \times \cancel{5}_{1}} = \frac{133}{4} = 33\frac{1}{4}$.

Or, $5)166\frac{1}{4}$

$33\frac{1}{4}$

EXPLANATION. — The mixed number $166\frac{1}{4}$ changed to an improper fraction equals $\frac{665}{4}$. Since a fraction is divided, etc. (Ex. 1). Or,

$1\frac{1}{4} \div 5 = \frac{\cancel{5}^{1}}{4 \times \cancel{5}_{1}} = \frac{1}{4}$.

Divide as in simple numbers. 5 is contained in $166\frac{1}{4}$ 33 times, with a remainder of $1\frac{1}{4}$; $1\frac{1}{4}$ equals $\frac{5}{4}$, which divided by 5 equals $\frac{1}{4}$; and $\frac{1}{4}$ added to the partial quotient 33 gives $33\frac{1}{4}$, the quotient required.

291. Rule to Divide a Fraction by an Integer.

I. *Divide the numerator of the fraction, or multiply the denominator, by the integer.*

II. *If the dividend is a mixed number, divide the integer and the fraction separately, and unite the results for the entire quotient required.*

Note. — If the mixed number is small, it is more convenient to change it to an improper fraction, and then divide.

Problems.

Divide —

1. $\frac{6}{7}$ by 3.
2. $\frac{7}{8}$ by 4.
3. $\frac{8}{9}$ by 6.
4. $\frac{10}{11}$ by 15.
5. $\frac{42}{47}$ by 28.
6. $\frac{75}{82}$ by 50.
7. $12\frac{3}{4}$ by 6.
8. $18\frac{7}{8}$ by 7.
9. $28\frac{5}{6}$ by 8.
10. $123\frac{7}{13}$ by 20.
11. $234\frac{8}{15}$ by 45.
12. $300\frac{3}{10}$ by 90.
13. Divide $\$22\frac{4}{5}$ by 4, by 6, by 8, by 10, by 12, by 18, by 48.
14. How much is $\frac{3}{11} \div 9$? $28\frac{3}{7} \div 9$? $100\frac{3}{100} \div 19$?

15. How much is $\frac{1}{5}$ of $\$1\frac{3}{12}$? $20\frac{7}{12}$ feet? $30\frac{5}{36}$ yards?

16. In 25 rods are $137\frac{1}{2}$ yards. How many yards in 1 rod?

17. If 9 tons of coal costs $\$61\frac{7}{8}$, what is the cost of 1 ton?

18. If one man can do a piece of work in $18\frac{3}{4}$ days, in what time can 3 men do it? 5 men? 7 men?

19. A flour-dealer's weekly sales amounted to $347\frac{3}{4}$ barrels. What were the average daily sales?

20. If $\frac{3}{7}$ of 21 yards of cloth cost $\frac{1}{2}$ of \$75, what is the cost of 1 yard?

CASE II.

To Divide an Integer by a Fraction.

1. How many fourths of a cake in 1 cake? In 2 cakes?

2. How many times $\$\frac{1}{4}$ in \$1? In \$2? In \$3? In \$4?

3. How many times is 1 contained in 2? How many times is $\frac{1}{2}$ contained in 2? $\frac{1}{3}$ in 2? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$?

4. 1 is how many times $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$?

5. Dividing by $\frac{1}{2}$ is the same as multiplying the dividend by what number? Dividing by $\frac{1}{5}$? $\frac{1}{7}$? $\frac{1}{9}$? $\frac{1}{11}$?

6. What is the quotient of 1 divided by $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$?

7. If one bottle holds $\frac{2}{3}$ of a quart of ink, how many bottles are needed to hold 6 quarts?

292. Analysis. — Since one bottle holds $\frac{2}{3}$ of a quart of ink, to contain 6 quarts, as many bottles are needed as the number of times $\frac{2}{3}$ of a quart are contained in 6 quarts.

Now 6 quarts equal $\frac{18}{3}$ of a quart; and $\frac{2}{3}$ is contained in $\frac{18}{3}$ as many times as 2 is contained in 18, which is 9 times. Hence, etc.

8. What is the quotient of $5 \div \frac{1}{3}$? Of $5 \div \frac{2}{3}$? The quotient of $5 \div \frac{2}{3}$ is what part of the quotient of $5 \div \frac{1}{3}$?

9. What is the quotient of $6 \div \frac{5}{7}$? It is the same as the dividend, 6, multiplied by what fraction?

10. Into how many pieces, each $2\frac{3}{4}$ feet long, can 22 feet of wire be cut?

293. Principle.

Multiplying the dividend multiplies the quotient, and dividing the dividend divides the quotient.

Written Exercises.

294. *Example.* — What is the quotient of $12 \div \frac{8}{9}$?

SOLUTION.

$$12 \div \frac{8}{9} = \frac{\overset{3}{\cancel{12}} \times 9}{\underset{2}{\cancel{8}}} = \frac{27}{2} = 13\tfrac{1}{2}.$$

EXPLANATION. — Since $12 \div 1 = 12$, $12 \div \frac{1}{9} = 9$ times 12, or 12×9; and $12 \div \frac{8}{9} = \frac{1}{8}$ of 12 $\times$ 9, or $\frac{12 \times 9}{8}$, which by cancellation equals $13\frac{1}{2}$, the quotient required.

295. *Problem.* — A farm of 69 acres was divided into lots of $2\frac{7}{8}$ acres each. How many lots were there?

SOLUTION.

$2\frac{7}{8}$ *acres* $= \frac{23}{8}$ *of an acre.*

$$69 \text{ acres} \div \frac{23}{8} \text{ acres} = \frac{\overset{3}{\cancel{69}} \times 8}{\underset{1}{\cancel{23}}} = 24, \text{ or } 24 \text{ lots.}$$

EXPLANATION. — Since each lot contained $2\frac{7}{8}$ acres, there were as many lots as the number of times $2\frac{7}{8}$ acres are contained in 69 acres.

$2\frac{7}{8}$ acres equal $\frac{23}{8}$ acres; 69 acres $\div \frac{1}{8}$ acre $= 69 \times 8$; 69 acres $\div \frac{23}{8} = \frac{69 \times 8}{23}$, which equals 24, the number of lots required.

296. Rule to Divide an Integer by a Fraction.

Multiply the integer by the denominator of the fraction, and divide the product by the numerator. The result will be the quotient required.

Note. — If the divisor is a mixed number, change it to an improper fraction, and then divide.

Problems.

Divide —

1. 6 by $\frac{4}{5}$.	5. 12 by $\frac{9}{10}$.	9. 55 by $6\frac{2}{7}$.	13. 100 by $25\frac{5}{9}$.
2. 7 by $\frac{6}{7}$.	6. 25 by $\frac{15}{16}$.	10. 81 by $7\frac{7}{8}$.	14. 575 by $50\frac{5}{9}$.
3. 8 by $\frac{8}{9}$.	7. 36 by $\frac{18}{25}$.	11. 90 by $8\frac{8}{9}$.	15. 757 by $75\frac{7}{10}$.
4. 9 by $\frac{6}{11}$.	8. 49 by $\frac{21}{32}$.	12. 98 by $9\frac{4}{5}$.	16. 5000 by $90\frac{7}{100}$.

17. Divide 25 feet by $\frac{6}{7}$, by $\frac{8}{9}$, by $1\frac{3}{5}$, by 10, by $10\frac{5}{7}$.

18. How much is $100 \div \frac{1}{20}$? $250 \div 5\frac{5}{17}$? $500 \div 20\frac{5}{11}$?

19. Find the quotient of $225 \div 8\frac{3}{4}$; $750 \div 15\frac{5}{8}$; $1000 \div 50\frac{11}{100}$; $1296 \div 60\frac{36}{145}$; $2592 \div 100\frac{6}{285}$.

20. Divide 100 feet of wire into pieces of $10\frac{1}{2}$ feet each.

21. At $\$\frac{7}{10}$ each, how many books can be bought for \$21? \$35? \$42? \$56? \$70? \$91? \$105?

22. Into how many fields can a farm of 230 acres be divided that each field shall contain $14\frac{3}{8}$ acres?

23. A builder paid a mason \$91 for labor at $\$3\frac{1}{2}$ a day. How many days did he work?

24. If wheat is worth $\frac{3}{5}$ of \$4 a bushel, how many bushels can be bought for \$19? For \$25? \$1000?

CASE III.

To Divide a Fraction by a Fraction.

1. How many times are 2 pints contained in 4 pints? 2 fifths of a ton in 4 fifths of a ton? 2 sixths in 4 sixths?

2. What is the quotient of 6 tenths $\div$ 3 tenths? $\frac{9}{10} \div \frac{3}{10}$?

3. What is the fractional unit of $\frac{8}{12}$? Of $\frac{4}{12}$? Are they similar, or dissimilar? How is the quotient of $\frac{8}{12} \div \frac{4}{12}$ found?

4. If the dividend and the divisor are similar fractions, how is the quotient obtained?

5. What is the quotient of $\frac{7}{8} \div \frac{3}{8}$? $\frac{9}{12} \div \frac{4}{12}$? $\frac{21}{15} \div \frac{9}{15}$?

6. At the rate of $\$\frac{7}{10}$ a bushel, how many bushels of corn can be bought for $\$2\frac{5}{8}$?

297. Analysis.—At the rate of $\$\frac{7}{10}$ a bushel, for $\$2\frac{5}{8}$ there can be bought as many bushels of corn as the number of times $\$\frac{7}{10}$ are contained in $\$2\frac{5}{8}$.

Since $\$\frac{7}{10}$ and $\$2\frac{5}{8}$ express fractional units that are unlike, they must first be changed to similar fractions having the least common

denominator, which is 40. $\$2\frac{1}{8}$, or $\$\frac{17}{8}$, equals $\$\frac{105}{40}$, and $\$\frac{7}{10}$ equals $\$\frac{28}{40}$. Now $\frac{28}{40}$ is contained in $\frac{105}{40}$ as often as 28 is contained in 105, which is $3\frac{1}{4}$ times. Hence, etc.

7. When the dividend and divisor are dissimilar fractions, how may the quotient be obtained?

8. Find the quotient of $\frac{3}{4} \div \frac{5}{6}$; $\frac{6}{7} \div \frac{8}{9}$; $\frac{5}{8} \div \frac{7}{8}$.

9. How often is $\frac{1}{3}$ contained in 1? In $\frac{1}{2}$? $\frac{2}{3}$ in 1? In $\frac{1}{2}$?

10. How often is $\frac{2}{3}$ contained in $\frac{3}{4}$?

298. Analysis. — Since 1 is contained in $\frac{3}{4}$ $\frac{4}{3}$ times, $\frac{1}{3}$ is contained in $\frac{3}{4}$ 3 times $\frac{3}{4}$ times, or $\frac{9}{4}$ times; and $\frac{2}{3}$ is contained in $\frac{3}{4}$ $\frac{1}{2}$ of $\frac{9}{4}$ times, or $\frac{9}{8}$ times. Hence, etc.

11. If the quotient of $\frac{2}{3} \div \frac{3}{4}$ is $\frac{8}{9}$, by which term of the divisor is the numerator of the dividend multiplied? The denominator?

12. How may the quotient of a fraction divided by a dissimilar fraction be found?

Definitions.

299. A ***Complex Fraction*** is a fraction which has a fraction as its numerator or its denominator or both.

Thus, $\frac{\frac{1}{2}}{4}$, $\frac{\frac{2}{3}}{4}$, $\frac{\frac{1}{2} \text{ of } \frac{2}{3}}{4\frac{1}{2}}$ are *complex fractions.*

Note. — A complex fraction is an expression of division of fractions, the numerator being the dividend, and the denominator being the divisor.

Written Exercises.

300. *Example* 1. — Divide $\frac{7}{8}$ by $\frac{4}{5}$.

SOLUTION.

$8 \times 5 = 40$ ***L. c. den.***

$\frac{7}{8} \div \frac{4}{5} = \frac{35}{40} \div \frac{32}{40} = 1\frac{3}{32}$.

Or, $\frac{7}{8} \div \frac{4}{5} = \frac{7}{8} \times \frac{5}{4} = \frac{35}{32} = 1\frac{3}{32}$.

EXPLANATION. — $\frac{7}{8}$ divided by $\frac{4}{5}$ is equivalent to $\frac{35}{40}$ divided by $\frac{32}{40}$, which gives $1\frac{3}{32}$, the quotient required. Or,

Since $\frac{7}{8}$ divided by 1 equals $\frac{7}{8}$, $\frac{7}{8}$ divided by $\frac{1}{5}$ must equal $\frac{7}{8} \times 5$, and $\frac{7}{8}$ divided by $\frac{4}{5}$ must equal $\frac{7}{8} \times \frac{5}{4}$, or $\frac{7}{8} \times \frac{5}{4}$, which equals $\frac{35}{32}$, or $1\frac{3}{32}$.

301. General Rule for Division of Fractions.

I. *Change integers and mixed numbers to improper fractions, and compound fractions to simple fractions.*

II. *Invert the terms of the divisor, and cancel factors common to the numerators and the denominators. Then multiply the numerators together for a new numerator, and the denominators together for a new denominator. The resulting fraction will be the quotient required.*

Note. — The problems under Case 1 and Case 2 may be solved according to the general rule by changing the integers to fractions.

Problems.

Divide —

1. $\frac{4}{5}$ by $\frac{2}{3}$.
2. $\frac{5}{6}$ by $\frac{4}{5}$.
3. $\frac{7}{8}$ by $\frac{6}{7}$.
4. $\frac{8}{9}$ by $\frac{7}{8}$.
5. $1\frac{4}{9}$ by $8\frac{2}{3}$.
6. $3\frac{6}{7}$ by $7\frac{5}{7}$.
7. $7\frac{7}{9}$ by $9\frac{3}{7}$.
8. $10\frac{7}{8}$ by $9\frac{2}{3}$.
9. $50\frac{5}{6}$ by $\frac{2}{3}$ of $36\frac{3}{5}$.
10. $78\frac{3}{8}$ by $\frac{4}{5}$ of $51\frac{9}{28}$.
11. $\frac{6}{7}$ of $57\frac{1}{6}$ by $91\frac{7}{8}$.
12. $\frac{9}{10}$ of $100\frac{5}{9}$ by $211\frac{1}{8}$

13. Divide $3\frac{1}{7}$ by 10; by $1\frac{2}{13}$; by $\frac{1}{2}$ of $8\frac{3}{4}$; by 8 times $42\frac{2}{5}$.

14. Divide $20\frac{5}{6}$ by 9; 100 by $9\frac{3}{8}$; $\frac{3}{8}$ of $6\frac{2}{5}$ by 30 $\times$ $3\frac{13}{24}$.

15. If the dividend is $3\frac{3}{5}$ times $10\frac{1}{2}$, and the divisor $\frac{5}{9}$ of $5\frac{5}{8}$, what is the quotient? If the dividend is $9\frac{3}{5} \times 11\frac{2}{3}$?

16. Divide $3\frac{5}{9}$ times $9\frac{3}{4}$ by $\frac{3}{4}$ of $\frac{9}{15}$ of $2\frac{2}{9}$; by $2\frac{1}{8} \times 24$.

17. If $7\frac{1}{2}$ bushels of wheat weigh $468\frac{1}{10}$ pounds, what is the average weight per bushel?

18. If 8 barrels of potatoes cost 6 times $\frac{4}{7}$ of $\$5\frac{3}{5}$, what is the price per bushel if each barrel contains $2\frac{1}{2}$ bushels?

302. *Example* 2. — Find the value of $\dfrac{6\frac{3}{7}}{9\frac{1}{3}}$

SOLUTION.

$$\frac{6\frac{3}{7}}{9\frac{1}{3}} = \frac{\frac{45}{7}}{\frac{28}{3}} = \frac{45}{7} \div \frac{28}{3}, \text{ and}$$

$$\frac{45}{7} \div \frac{28}{3} = \frac{45}{7} \times \frac{3}{28} = \frac{135}{196}.$$

EXPLANATION. — Since the complex fraction is an expression of division, change the dividend and the divisor to improper fractions. Then invert the terms of the denominator, or divisor, and cancel factors, etc. (300).

Find the value of—

19. $\frac{12\frac{2}{3}}{19}$, or $12\frac{2}{3} \div 19$.

20. $\frac{45}{10\frac{4}{5}}$, or $45 \div 10\frac{4}{5}$.

21. $\frac{20\frac{4}{5}}{6\frac{1}{2}}$, or $20\frac{4}{5} \div 6\frac{1}{2}$.

22. $\frac{6\frac{2}{7}}{\frac{1}{2}\frac{1}{8}}$; $\frac{\frac{14}{15}}{12\frac{6}{7}}$; $\frac{7\frac{3}{7}}{8\frac{2}{3}}$; $\frac{8\frac{5}{9}}{\frac{2}{9}\text{ of }19\frac{4}{5}}$.

23. $\frac{10\frac{5}{7}}{13\frac{1}{8}}$; $\frac{3\frac{1}{2}}{\frac{2}{3}\text{ of }\frac{6}{7}}$; $\frac{\frac{3}{5}\text{ of }2\frac{3}{11}}{8\frac{2}{5}}$.

24. $\frac{14\frac{2}{3}}{17\frac{2}{7}}$; $\frac{4\frac{9}{10}}{\frac{3}{7}\text{ of }8\frac{2}{5}}$; $\frac{2\frac{4}{7} \times \frac{8}{9}}{\frac{3}{10}\text{ of }9\frac{3}{8}}$.

25. Change to a simple fraction $\frac{50\frac{1}{10}}{20\frac{7}{8}}$; $\frac{4\text{ times }3\frac{3}{8}}{\frac{3}{7}\text{ of }70\frac{7}{10}}$.

26. At $\frac{3}{4}$ of $\$1\frac{60}{100}$ per yard, how many yards of cloth can be bought for $\frac{5}{6}$ of $\$10\frac{2}{25}$? For $\frac{7}{12}$ of $\$14\frac{2}{5}$?

27. If $180\frac{1}{2}$ bushels of corn are raised on $5\frac{1}{3}$ acres, what is the average yield per acre?

28. What fraction multiplied by $4\frac{1}{2}$ times $2\frac{2}{9} = 5\frac{3}{4} \times 6\frac{1}{2}$?

29. If a lot of ground containing $2810\frac{1}{4}$ square feet is worth $1000, what is the price per foot?

30. How many lengths of carpet, each containing $6\frac{2}{3}$ square yards, are needed to cover $73\frac{1}{3}$ square yards of floor?

Review Problems in Fractions.

Find the value of—

1. $\frac{3}{7} + \frac{4}{9} \times \frac{7}{10} - \frac{7}{11}$.

2. $\frac{8}{9} + \frac{5}{6} \times \frac{36}{35} \div \frac{10}{11}$.

3. $\frac{1}{1}\frac{3}{5} + (\frac{9}{26} \times \frac{39}{40}) \div \frac{7}{80}$.

4. $(\frac{9}{20} \div \frac{8}{15}) \times \frac{9}{14} - \frac{2}{21}$.

5. $8\frac{2}{3} - \frac{2}{5}\text{ of }3\frac{1}{3} \times (4\frac{1}{5} + \frac{9}{10})$.

6. $(4\frac{2}{7} \times \frac{8}{15} + 8\frac{1}{3}) + \frac{1}{21}\text{ of }\frac{35}{56}$.

7. $(7\frac{3}{5} \div \frac{8}{19}) + 8\frac{1}{3} \times \frac{3}{5}\text{ of }\frac{8}{9}$.

8. $(8\frac{4}{5} - 7\frac{3}{4}) \div \frac{3}{8}\text{ of }10\frac{2}{3} + 17$.

9. Find the sum of $3\frac{1}{2} \times \frac{6}{7}$ and $8\frac{2}{3} \div 7\frac{4}{5}$. The difference.

10. From $10\frac{4}{5} \div 12\frac{6}{7}$ take $13\frac{3}{10}$ times $\frac{1}{19}$. Find the sum?

11. Find the quotient of $10\frac{1}{20}$ times $\frac{10}{87}$ divided by $50\frac{5}{26} - 40\frac{5}{39}$. To the quotient add $7\frac{1}{10}$ times $14\frac{2}{7}$.

12. Multiply the sum of $13\frac{2}{21}$ and $17\frac{13}{14}$ by the quotient of $40\frac{5}{13} \div 4\frac{1}{26}$. Multiply the product by $3\frac{7}{11}$.

13. Divide the sum of $3\frac{1}{10}$, $8\frac{7}{15}$, and $10\frac{9}{20}$ by the difference between $29\frac{2}{3}\frac{9}{0}$ and $7\frac{1}{2}\frac{8}{0}$.

14. What number multiplied by $\frac{1}{2}$ of $\frac{3}{10}$ of $\frac{3}{8}$ gives 360?

15. What number is that which, multiplied by $\frac{3}{7}$ of $\frac{2}{10}$, gives $100\frac{7}{8}$? Gives $250\frac{3}{11}$? $500\frac{9}{25}$? $1000\frac{9}{140}$?

16. By what must you multiply the difference between $100\frac{1}{30}$ and $27\frac{7}{10}$, that the product may be $207\frac{7}{9}$?

17. The product of three factors is $127\frac{2}{4}\frac{9}{8}$, and two of them are $9\frac{7}{12}$ and $15\frac{5}{23}$. What is the third?

18. The divisor is $9\frac{10}{11}$, the quotient is $15\frac{21}{25}$, and the remainder is $9\frac{3}{20}$. What is the dividend?

19. Find the sum of 20 and $\frac{1}{20}$, and their difference. Divide the sum by the difference, the difference by the sum, and find the difference of the two quotients.

20. $\frac{3}{4}$ of $\frac{6}{7}$ of John's money is \$900. How much is $\frac{4}{5}$ of his money? $7\frac{1}{2}$ times his money? $7\frac{1}{2}$ times $\frac{4}{5}$ of it?

21. If $\frac{7}{8}$ of a yard of broadcloth cost $\$6\frac{4}{7}$, how many yards can be bought for $\$215\frac{5}{8}$? For $\$1000\frac{50}{100}$?

22. If $7\frac{5}{8}$ tons of hay are worth $\$150\frac{8}{8}$, how much are $22\frac{7}{8}$ tons worth? $44\frac{7}{16}$ tons? $4\frac{1}{2}$ times $11\frac{7}{16}$ tons?

23. If 4 yards of cloth cost $\$7\frac{3}{8}$, 7 yards will cost how much more than $\$12\frac{1}{2}$? How much less than $\$15\frac{7}{7}$?

24. A man is 45 years old; $\frac{4}{5}$ of his age is $\frac{6}{7}$ of his wife's age. How old is his wife? Find the sum of their ages.

25. What is the average weight of three turkeys that weigh $8\frac{3}{8}$ pounds, $15\frac{9}{16}$ pounds, and $14\frac{3}{4}$ pounds?

26. I own $\frac{7}{9}$ of a ship, and John owns $\frac{9}{45}$. How many times as much as he do I own? Both own what part?

27. A merchant put $\frac{1}{3}$ of his money in one bank, $\frac{1}{4}$ in another, and \$700 in a third. How much money had he?

28. A captain owned $\frac{5}{8}$ of a vessel, and sold $\frac{2}{3}$ of his share for \$4800. At this rate, what was the whole ship worth?

29. A man owned $\frac{7}{8}$ of a factory worth \$8000. He sold

$\frac{2}{7}$ of his share to one man, and $\frac{2}{5}$ of the remainder to another. What was his part then worth?

30. A butcher killed 4 steers weighing $1025\frac{1}{8}$ pounds, $927\frac{3}{10}$ pounds, $898\frac{4}{5}$ pounds, and 1400 pounds. What was the average weight?

31. If one train of cars runs $225\frac{3}{4}$ miles in 10 hours, and another runs $378\frac{5}{8}$ miles in 18 hours, which runs the faster per hour, and how much?

32. A part of a 50-acre farm was sold for $\$1200\frac{75}{100}$, and the balance for $\$1750\frac{7}{8}$. What was the average selling price per acre?

33. The sum of $3\frac{7}{30}$ and $\frac{4\frac{2}{7}}{7\frac{7}{9}}$ is how many times their difference? Multiply their sum by their product.

34. Find the sum of $\frac{\frac{1}{2}+\frac{3}{4}}{\frac{49}{50}-\frac{7}{10}}$ and $7\frac{1}{2}$ times $\frac{1\frac{3}{5}\times\frac{5}{26}}{8\frac{1}{10}-\frac{81}{100}}$.

35. Multiply $8\frac{1}{2}$ times $\frac{3\frac{3}{4}}{4\frac{2}{5}}+\frac{\frac{2}{3}\text{ of }7\frac{1}{5}}{\frac{3}{5}+\frac{3}{7}}$ by their difference.

Review.

1. State the principles of ***Addition of Fractions.*** Repeat the rule. In what two ways may mixed numbers be added? How must the final result of any operation in fractions be expressed?

2. State the principles of ***Subtraction of Fractions.*** Repeat the rule. In what two ways may mixed numbers be subtracted?

3. In what two ways may the value of a fraction be multiplied? State the principle of multiplying a fraction by an integer. Repeat the rule. In what two ways may a mixed number be multiplied by an integer? State the principle of multiplying an integer by a fraction. Repeat the rule. In what two ways may an integer be multiplied by a mixed number?

4. Define ***Simple Fraction.*** What is a compound fraction? Of what is a compound fraction composed? Repeat the general rule for multiplication of fractions. Tell how a compound fraction is changed to a simple one.

5. In what two ways may the value of a fraction be divided?

State the principle of dividing a fraction by an integer. Repeat the rule. In what two ways may a mixed number be divided by an integer? State the principle of dividing an integer by a fraction. Repeat the rule. How is an integer divided by a mixed number?

6. Define ***Complex Fraction.*** Of what is a complex fraction an expression? Repeat the general rule for division of fractions. Explain how to change a complex fraction to a simple one.

SECTION XIV.

DECIMAL FRACTIONS.

1. If a cake is divided into 10 equal parts, what part of the cake is one of the parts? What part of the cake are 2 of the parts? 3 of the parts? 5? 7? 9?

2. If a unit is divided into 10 equal parts, what are the parts called?

3. How much is $\frac{1}{10}$ of 1? $\frac{2}{10}$ of 1? $\frac{3}{10}$? What is the fractional unit of 5 tenths? 6 tenths? $\frac{7}{10}$? $\frac{8}{10}$?

4. What part of 1 is 1 tenth? How many tenths in 1?

5. If 1 tenth of a dollar is divided into 10 other equal parts, what part of a dollar is each part? What part of a dollar are 2 of the parts? 3 of the parts? 5?

6. If a unit is divided into 10 equal parts, and each tenth into 10 equal parts, or the unit into 100 equal parts, what are the parts called?

7. How much is $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{3}{10}$ of $\frac{1}{10}$? $\frac{9}{10}$ of $\frac{1}{10}$? What is the fractional unit of 2 hundredths? $\frac{4}{100}$? $\frac{6}{100}$?

8. What part of 1 tenth is 1 hundredth? What part of 1 is 1 hundredth? How many hundredths is 1 tenth? In 1?

9. If 1 hundredth of a block is divided into 10 other equal parts, what part of the block is each part? What part of the block are 2 of the parts? 4 of the parts? 6?

10. If a unit is divided into 100 equal parts, and each hundredth into 10 equal parts, or the unit into 1000 equal parts, what are the parts called?

11. How much is $\frac{1}{10}$ of $\frac{1}{100}$? $\frac{3}{10}$ of $\frac{1}{100}$? $\frac{7}{10}$ of $\frac{1}{100}$? What is the fractional unit of 3 thousandths? $\frac{7}{1000}$? $\frac{9}{1000}$?

12. What part of 1 hundredth is 1 thousandth? What part of 1 is 1 thousandth? How many thousandths in 1 hundredth? In 1?

Definitions.

303. A ***Decimal Fraction*** is a fraction whose denominator is 10, 100, or 1000, etc.

Thus, *1 tenth, 5 hundredths,* $\frac{5}{10}$, $\frac{7}{100}$, $\frac{9}{1000}$ are *decimal fractions.*

Decimal fractions are so called because they are decimal divisions of a unit.

304. Decimal Fractions are commonly called ***Decimals.***

Notation and Numeration of Decimals.

305. Since in decimals the values of fractional units decrease by the divisor 10, and increase by the multiplier 10, as integers do, they are governed by the same principles as are integers, and may be expressed in the same manner by extending the orders of decimal units below ones.

306. In naming decimals ***one tenth of one*** is called ***one tenth.***

Tenths are written: —

1 tenth,	.1	*4 tenths,*	.4	*7 tenths,*	.7
2 tenths,	.2	*5 tenths,*	.5	*8 tenths,*	.8
3 tenths,	.3	*6 tenths,*	.6	*9 tenths,*	.9

The ***Decimal Point*** is a period (.) placed before tenths.

307. When a decimal is expressed by a single figure, it denotes tenths.

308. In naming decimals ***one tenth of one tenth*** is called ***one hundredth.***

Hundredths are written: —

1 hundredth, .01	*4 hundredths, .04*	*7 hundredths, .07*
2 hundredths, .02	*5 hundredths, .05*	*8 hundredths, .08*
3 hundredths, .03	*6 hundredths, .06*	*9 hundredths, .09*

309. To express a number of hundredths, or a number of tenths and hundredths, two decimal figures are needed.

310. When a decimal is expressed by two figures, the first figure to the right of the decimal point denotes tenths, and the second figure denotes hundredths.

1. How many tenths and how many hundredths are in the decimal .13? In .25? .37? .59? .68? .06? .04?

311. In naming or reading decimals expressed by two figures, the tenths and the hundredths are read together as hundredths.

Thus, .34 is named or read thirty-four hundredths. For .34 is 3 tenths 4 hundredths; but 3 tenths equal 30 hundredths, and 30 hundredths + 4 hundredths equal 34 hundredths.

312. In naming decimals ***one tenth of one hundredth*** is called ***one thousandth.***

Thousandths are written: —

1 thousandth,	*.001*	*5 thousandths,*	*.005*
2 thousandths,	*.002*	*6 thousandths,*	*.006*
3 thousandths,	*.003*	*7 thousandths,*	*.007*
4 thousandths,	*.004*	*8 thousandths,*	*.008*

9 thousandths, .009.

313. To express a number of thousandths, or of hundredths and thousandths, or of tenths, hundredths, and thousandths, three decimal figures are needed.

314. When a decimal is expressed by three figures, the third figure to the right of the decimal point denotes thousandths, and the decimal is named or read *thousandths.*

2. How many tenths, how many hundredths, and how many thousandths are in the decimal .102? In .024? .350? .004? .506? .678? .007?

315. In naming decimals ***one tenth of one thousandth*** is called ***one ten-thousandth.***

316. To express a number of ten-thousandths, four figures are needed.

317. When a decimal is expressed by four figures, the fourth figure denotes ten-thousandths, and the decimal is read ten-thousandths.

3. Name each order of decimal units in the following decimals, and read each decimal: .0009; .0098; .0918; .9182; .8027; .7306; .6004; .0505; .0430.

318. In naming decimals ***one tenth of one ten-thousandth*** is called ***one hundred-thousandth.***

319. To express a number of hundred-thousandths, five figures are needed.

320. When a decimal is expressed by five figures, the fifth figure denotes hundred-thousandths, and the decimal is read hundred-thousandths.

4. Name each order of decimal units in the following decimals, and read each decimal: .00009; .00098; .00807; .07065; .70605; .60054; .05004.

321. In naming decimals ***one tenth of one hundred-thousandth*** is called ***one millionth.***

322. To express a number of millionths, six figures are needed.

323. When a decimal is expressed by six figures, the sixth figure denotes millionths, and the decimal is read millionths.

5. Tell how many millionths are in the following decimals, name each order of decimal units, and read each decimal: .000001; .010009; .900803; .090102; .030041.

324. A ***Decimal Unit*** is one of the equal decimal parts into which the unit 1 is divided.

Thus, $\frac{1}{10}$ is a decimal unit; so, also, is 1 hundredth, 1 thousandth, etc.

Note.—The first order *above* ones is *tens*, and the first *below* is *tenths;* the second *above* is *hundreds*, and the second *below* is *hundredths*, etc.

The orders of integral units above ones always end with *ns* or *ds*, while the corresponding orders of decimal units below ones end with *nths* or *dths*.

Table of Orders.

Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	ONES.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.
7	6	5	4	3	2	1.	2	3	4	5	6	7
INTEGERS.							DECIMALS.					

The ninth order of decimal units is *billionths*, the twelfth is *trillionths*.

325. Decimal fractions may be expressed in figures in two ways:—

1. As *common fractions* with the denominator expressed; as $\frac{1}{10}$, $\frac{3}{100}$, $\frac{5}{1000}$, etc.

2. In *decimal notation* with the denominator omitted, but indicated by the orders of units expressed by the numerator; as *.1*, *.03*, *.005*, etc.

326. Since in decimals as in integers a figure has a simple and also a local value, and since the local value of a figure depends upon the decimal place in which it stands,

any change in the position of the numerator must affect the value of the decimal. Hence,

327. Principles of Decimals.

I. *Ten units of any decimal order equal one decimal unit of the next higher order.*

II. *The value of a decimal figure is decreased tenfold for every place it is removed from the left toward the right.*

III. *The value of a decimal figure is increased tenfold for every place it is removed from the right toward the left.*

IV. *The value of a decimal is not changed by annexing ciphers to, or omitting ciphers from, the right of the decimal.*

328. Rule for Decimal Notation.

Write the numerator of the decimal as if it were an integer, and so place the decimal point that the right-hand figure shall stand in the place denoted by the denominator of the decimal.

Note. — If the numerator does not fill all the place denoted by the denominator, supply the deficiency by prefixing ciphers.

329. Rule for Decimal Numeration.

I. *Numerate from the right toward the decimal point to determine the numerator, and from the decimal point toward the right to determine the denominator or name of the decimal.*

II. *Read the decimal as if it were an integer, and give it the name of the right-hand decimal order.*

Exercises.

Express in figures *seventy-five thousandths.*

330. Model. — Since in 75 thousandths the name or denominator is *thousandths*, the decimal must consist of three decimal places.

Write 75 as an integer, and prefix one cipher, that the right-hand figure shall stand in the third or thousandths' place, and place the decimal point before tenths, giving *.075*, the expression required.

Express in words, or numerate and read, the decimal *.01008.*

331. Model. — Begin at the right and separate the decimal into periods, as in integers, giving *1003*. The numerator is *one thousand and three.*

Begin at tenths to numerate the decimal orders: tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths. The name of the decimal, or the denominator, is *hundred-thousandths.*

The decimal is read *one thousand and three hundred-thousandths.*

1. Express in decimal form: —

1. $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$.	4. $\frac{10}{100}$, $\frac{25}{100}$.	7. $\frac{214}{1000}$, $\frac{225}{1000}$.	10. $10\frac{8}{10}$.
2. $\frac{5}{10}$, $\frac{7}{10}$, $\frac{9}{10}$.	5. $\frac{9}{100}$, $\frac{19}{100}$.	8. $\frac{53}{1000}$, $\frac{79}{1000}$.	11. $9\frac{2}{100}$.
3. $\frac{5}{100}$, $\frac{7}{100}$.	6. $\frac{8}{100}$, $\frac{99}{100}$.	9. $\frac{4}{1000}$, $\frac{7}{1000}$.	12. $7\frac{9}{1000}$.

13. $10\frac{1}{10}$, $\frac{1}{100}$, $100\frac{1}{100}$, $\frac{3}{1000}$, $3000\frac{8}{1000}$, $3003\frac{8}{1000}$.

14. $\frac{500}{1000}$, $\frac{5}{100000}$, $\frac{505}{1000000}$, $5000\frac{5}{100000}$, $\frac{5005}{100000}$.

15. 6 tenths; 6 hundredths; 6 thousandths; 6 ten-thousandths.

16. Four hundred-thousandths; five and five millionths.

17. 100 and 1 thousandth; 101 thousandths; two hundred and two hundred-thousandths; one millionth.

18. 100 thousandths; 1 hundred-thousandth; 100 hundred-thousandths; 101 thousandths; 100 and 1 thousandth.

19. One hundred thousand and 1 hundred-thousandth; 100 million; 1 hundred-millionth; one hundred one millionths.

20. Two thousand three ten-thousandths; two thousand and three ten-thousandths; thirty thousand three ten-millionths.

2. Express in words, or numerate: —

1. .8; .1; .9.	4. 9.02; 7.09.	7. 1.025; 5.005.	10. .90715; .08025.
2. 7.1; 9.5.	5. 81.5; 8.15.	8. 50.05; 500.5.	11. .00713; .00024.
3. .01; .03.	6. .815; .91.	9. .8125; .0725.	12. .00006; 5.00005.

13. .002; 200.002; .0010; 10.010; 900.0007; 7005.7005.

14. 90800.7; 9080.07; 908.007; 90.8007; 9.08007; .908007.

15. .060024; .005003; .000304; .000007; .0030075; .0004006.

16. 10.1; 100.01; 1000.001; .200; .00002; 300.00003; 3.000003.

17. 400.004; .404; .00404; 400000.00004; 1000000.000001.

18. .0002; 30000.030; 404.00404; .0200304; .01090208.

19. 3000010.0000003; .090102082; .008003007; .000070009.

20. 80000.800008; 70.0007007; 6000.0000606; 500.500505.

Review.

1. What is a ***Decimal Fraction?*** Give an example. Why are decimal fractions so called? In integers, how does the value of orders increase and decrease? In decimals, how does the value of the fractional units increase and decrease?

2. What is meant by ***Numeration of Decimals?*** In naming decimals, what is one tenth of one called? One tenth of one tenth? One tenth of one hundredth? One tenth of one thousandth? One tenth of one millionth? How many tenths make one? What make one tenth? One hundredth? One thousandth? One ten-thousandth? One hundred-thousandth? Repeat the rule.

3. What is meant by ***Notation of Decimals?*** In what two ways may decimals be expressed? When a decimal is expressed in decimal form by one figure, what does it denote? When expressed by two decimal figures, what does each denote? By three decimal figures? By five? By seven? How many decimal figures are needed to express tenths? Hundredths? Thousandths? Millionths? Ten-thousandths? Ten-millionths? Repeat the rule.

4. What is a ***Decimal Unit?*** Name the first order of decimal units; the fourth; the seventh; the second; the fifth; the eighth; the third; the sixth; the ninth. In what place are tenths? Ten-thousandths? Ten-millionths? Hundredths? Hundred-thousandths? Hundred-millionths? Hundredths? Thousandths? Millionths? What two values has a decimal figure? Upon what does the local value of a decimal figure depend? State the general principles of decimals.

SECTION XV.

ADDITION OF DECIMALS.

1. How many ones in 5 ones and 7 ones? Tens and ones?

2. How many tenths in 3 tenths and 4 tenths? $\frac{4}{10}$ and $\frac{5}{10}$?

3. What is the sum of $\frac{5}{10}$ and $\frac{9}{10}$? Of .7 and .8?

4. Find the sum of 5 hundredths and 10 hundredths. Of $\frac{4}{100}$ and $\frac{15}{100}$; of .05 and .75; .12 and .88.

5. Can 2 tens and 7 ones be directly added? 2 tenths and 7 hundredths? $\frac{3}{10}$ and $\frac{8}{100}$? Why?

6. What is the sum of $\frac{7}{10}$ and $\frac{45}{100}$? Of .5 and .07?

7. What is the sum of .05 and .09? How many hundredths and tenths? Of .07 and .08?

8. Find the sum of 8 and $\frac{1}{2}$. Of 7 and $\frac{5}{10}$? Of 6 and .6? 5 and .07? .4 and .08? 3 + .9 + .05?

9. Find the sum of $\frac{3}{10}$, $\frac{5}{100}$, and $\frac{6}{1000}$. Of .4, .06, .007.

10. What kind of fractions only can be added? When only can orders of decimal units be added?

332. Principle.

Only similar orders of decimal units can be added.

Written Exercises.

333. *Example.* — Find the sum of 7.8, 15.725, 8.08, and .8765.

SOLUTION.

7.8
15.725
8.08
.8765
32.4815

EXPLANATION. — Since only like orders of decimal units can be added, write the parts so that figures expressing units of the same order shall stand in the same column, and the decimal points in a column.

Begin at the lowest order at the right, and add as in integers, placing the decimal point before tenths in the sum, giving *32.4815*, the sum required.

334. Rule for Addition of Decimals.

I. *Write the numbers so that units of the same order shall stand in the same column.*

II. *Add as in addition of integers, and place the decimal point before the order of tenths in the sum.*

Problems.

(1)	(2)	(3)	(4)	(5)
.97	*70.075*	*17.0075*	*$73.25*	*10.875 miles.*
.089	*.087*	*.07*	*75.365*	*9.0705 "*
.85	*10.9*	*7.825*	*.0075*	*98.678 "*
.087	*.008*	*.91*	*23.29*	*8.3789 "*
.85	*9.257*	*80.8*	*8.8*	*87.987*

Find the sum of —

6. 82.765 + 8000 + .0075 + 87.1 + .00876 + 30.02.

7. $27.50 + $.125 + $300 + $98.625 + $17625.50.

8. 30 miles + 8.025 miles + 1000 miles + 350.0025 miles.

9. Add together 8 tenths, 25 hundredths, 78 ten-thousandths, 5 millionths, and 2004 hundred-thousandths.

10. Find the sum of 24 hundreds, 24 tens, 24 ones, 24 tenths, 24 hundredths, and 24 thousandths.

11. A coal-dealer sold in four weeks 75.75 tons, 125.8 tons, 90.0625 tons, and 50.825 tons. How many tons in all?

12. A grocer bought a ton of coal for $6.75, a barrel of flour for $8.125, a chest of tea for $28.25, and a hogshead of molasses for $22.375. What did all cost?

SECTION XVI.

SUBTRACTION OF DECIMALS.

1. How many tenths are 7 tenths less 4 tenths? $\frac{8}{10}$ less $\frac{3}{10}$?

2. What is the difference between $\frac{9}{10}$ and $\frac{4}{10}$? .7 and .2?

3. From 15 hundredths take 5 hundredths. From $\frac{25}{100}$ take $\frac{6}{100}$. From .75 take .05. From 1 take .25.

4. From $\frac{75}{1000}$ take $\frac{5}{1000}$. From .025 take .007.

5. What is $\frac{5}{10}$ less $\frac{5}{100}$? $\frac{6}{100} - \frac{6}{1000}$? .7 — .07?

6. What is the difference between .09 and .009? 2 and .02?

7. Find the value of $\frac{9}{10} - .4$; .08 — .008.

8. When only can one decimal fraction be taken from another?

335. Principle.

Only similar orders of decimal units can be taken one from another.

Written Exercises.

336. *Example.* — From 8.75 take .0875.

Solution.		
8.7500	8.75	
.0875	Or, .0875	
8.6625	8.6625	

Explanation. — Since only similar orders of decimal units can be taken one from another, write the numbers so that figures expressing units of the same order shall stand in the same column.

Annex two ciphers to the minuend to make the number of decimal places equal to the subtrahend; or regard them as annexed, and subtract as in integers, placing the decimal point before the order of tenths in the difference, giving *8.6625*, the difference required.

337. Rule for Subtraction of Decimals.

I. *Write the subtrahend under the minuend so that units of the same order shall stand in the same column.*

II. *Subtract as in subtraction of integers, and place the decimal point before the order of tenths in the remainder.*

Problems.

(1)	(2)	(3)	(4)	(5)
.57	70.7025	6.985	8.625 tons.	$20.125
.1392	.98	1.0875	.75 "	15.1875

Find the remainder of —

6. 100.01 — 99.9; 500.5 — 5.05; 20.02 — .002.

7. 7000 — .007; 6 — .006; 50.057 — .00075.

8. $300.25 — $75.625; 200.8 miles — 150.875 miles.

9. If the minuend is 36 feet, and subtrahend is 20.0025 feet, what is the remainder?

10. From 5 hundred take 5 hundredths. From five thousand and five thousandths take five millionths.

11. I sold a horse for $250, which was $25.75 more than I paid for him. What did he cost me?

12. A housekeeper's bill at a grocer's store was $11.75. What change should she receive from two 10-dollar bills?

13. $2 + .002 - .0008$; $(300 - .03) + (2.99 + .0075) = ?$

14. To the difference between 5000 and .005 add .00005.

15. A man owed \$1000, and paid at one time \$300.75, and at another \$500.12½. What did he then owe?

16. If a pair of horses cost \$350.75, and a carriage \$175.50, and are all sold for \$650.12½, what is the gain?

SECTION XVII.

MULTIPLICATION OF DECIMALS.

1. How many ones are 4 ones taken 2 times? Taken 1 time? $\frac{1}{2}$ of 1 time? $\frac{1}{10}$ of 1 time?

2. How many tenths are 4 times 2 tenths? 3 times $\frac{3}{10}$?

3. How much is 2 times $\frac{4}{10}$? 3 times .2? $.4 \times 2$?

4. What is the product of $\frac{4}{10}$ taken 2 times? 1 time?

5. What is the product of $\frac{4}{100} \times 3$? Of $.03 \times 6$?

6. What is the product of $\frac{2}{10} \times \frac{3}{10}$? $\frac{9}{10} \times \frac{8}{10}$? $.4 \times .2$?

7. What is the product of $\frac{4}{10} \times \frac{2}{100}$? $.05 \times .3$? $.2 \times .04$?

8. What denominator is produced by multiplying tenths by ones? Hundredths by ones? Tenths by tenths? Tenths by hundredths, or hundredths by tenths?

9. How many ciphers are in the product of the denominators of any two decimal fractions?

10. How many decimal places are in the product of tenths by ones? Hundredths by ones? Tenths by tenths? Tenths by hundredths, or hundredths by tenths?

338. Principle.

The number of decimal places in the product equals the number of decimal places in both factors.

Written Exercises.

339. *Example.* — Multiply 9.8 by .06.

SOLUTION.
$$\begin{array}{r} 9.8 \\ .06 \\ \hline .588 \end{array}$$

EXPLANATION. — Begin with the lowest order of the multiplier, and multiply each order of the multiplicand separately, as in integers.

6 hundredths times 8 tenths are 48 thousandths, or 4 hundredths 8 thousandths; write 8 thousandths, etc.

6 hundredths times 9 ones are 54 hundredths; 54 hundredths and 4 hundredths are 58 hundredths, or 5 tenths 8 hundredths, etc.

Prefix the decimal point, giving *.588*, the product required.

Or, since tenths multiplied by hundredths produce thousandths, the product must contain three decimal places, or as many decimal places as are in both factors, giving *.588*, the product required.

340. Rule for Multiplication of Decimals.

Multiply as if the numbers were integers, and from the right hand of the product point off as many figures for decimals as there are decimal places in both factors.

Note 1. — If there are not so many figures in the product as there are decimal places in both factors, supply the deficiency by prefixing ciphers.

2. To multiply by 10, 100, 1000, etc., remove the decimal point in the multiplicand as many places to the right as there are ciphers in the multiplier.

Problems.

(1)	(2)	(3)	(4)	(5)
78.54	*29.384*	*5.0821*	*$80.565*	*18.7625 feet.*
.87	*75.6*	*40.75*	*5.006*	*5400*

Find the product of —

6. 25 times $2.25; 56 times 8.175; 875 times .008.

7. .5 of $9.75; .18 of 5.25 tons; .75 of 100.235.

8. 91.82 × .135; .7364 × 50; .0012 × .12; .009 × .008.

9. 900.125 miles × $8\frac{1}{2}$; 70.0876 miles × $8.5\frac{1}{2}$.

10. Multiply 5 million and 5 millionths by 5 and 5 millionths; by five thousand five hundred-thousandths.

11. If the multiplicand is four hundred seventy-five hundred-thousandths, and the multiplier is four hundred and seventy-five hundred-thousandths, what is the product?

12. What is the product of .7 × .07 × .007? 700 × 70 × .7? 100.01 × .01 × .001? 10000 × .0001 × 1.0001?

13. If a bushel of wheat weighs 60.75 pounds, what is the weight of 10 sacks, each containing 3 bushels?

14. If a horse can eat 1.3125 bushels of corn in a week, how many bushels are needed for 15 horses for 13 weeks?

15. Find the sum of .004 of \$5000 + $\overline{\$.025 \times 25000}$.

16. Multiply 2000 by .002. To their product add their sum. From their difference take their product.

17. A grocer bought a barrel of flour, 196 pounds, at \$.04½ a pound, and retailed it at \$.05¼ a pound. What did he gain?

18. If a merchant buys a case of silk containing 16 pieces, at \$53.25 a piece, and pays \$1.25 for the box, what is the whole cost? What is gained by selling at \$53.37½ a piece?

SECTION XVIII.

DIVISION OF DECIMALS.

1. How much is ½ of 8 tenths? ¼ of 12 hundredths?

2. What is ½ of $\frac{6}{10}$? ¼ of $\frac{12}{100}$? ⅕ of $\frac{35}{1000}$?

3. How much is ¼ of .8? ⅓ of .6? ¼ of .24? ⅕ of .035?

4. How many times are 2 tenths contained in 8 tenths?

5. Divide $\frac{4}{10}$ by $\frac{4}{100}$; $\frac{5}{100}$ by $\frac{5}{1000}$; .6 by .06; .08 by .008.

6. What is the quotient of 2.4 divided by 8?

341. Analysis. — In 1 are 10 tenths, and in 2 are two times 10 tenths, which are 20 tenths; 20 tenths and 4 tenths are 24 tenths; ⅛ of 24 tenths is 3 tenths. Hence, etc.

7. Divide 3 by .6; .3 by 6; .03 by .6; .006 by .03.

8. The product of two factors is .015, and one of the factors is .5. What is the other factor?

9. The dividend is the product of what two factors? The dividend contains as many decimal places as what two factors?

10. Of what order is the quotient, if ones are divided by ones? Tenths by ones? Tenths by tenths? Hundredths by ones? Hundredths by tenths? Thousandths by tenths?

342. Principle.

The number of decimal figures in the quotient equals the number of decimal places in the dividend, less the number of decimal places in the divisor.

Written Exercises.

343. *Example* 1. — Divide 3.015 by 9.

SOLUTION.

9)*3.015*
.335

EXPLANATION. — 3.015 equals 3015 thousandths; and $\frac{1}{9}$ of 3015 thousandths is 335 thousandths, or .335, the quotient required. Or,

Divide as in division of integers; and since the dividend contains three decimal places, and the divisor has no decimal places, the quotient must have three decimal places.

344. *Example* 2. — Divide .625 by 2.5.

SOLUTION.

2.5).*625*(*.25*
50
125
125

EXPLANATION. — Divide as in division of common fractions (301).

$.625 = \frac{625}{1000}$, and $2.5 = \frac{25}{10}$; and $.625 \div 2.5 = \frac{625}{1000} \div \frac{25}{10} = \frac{625}{1000} \times \frac{10}{25}$, which equals $\frac{25}{100}$, or .25, the quotient required. Or,

Divide as in division of integers; and since the dividend contains three decimal places, and the divisor one decimal place, the quotient must have two decimal places.

345. Rule for Division of Decimals.

Divide as if the numbers were integers, and from the right hand of the quotient point off as many figures for decimals as

the number of decimal places in the dividend exceeds the number of decimal places in the divisor.

Note 1. — If the number of figures in the quotient is less than the difference between the number of decimal places in the dividend and the number in the divisor, supply the deficiency by prefixing ciphers.

2. If the dividend has not so many decimal places as the divisor, make the number equal by annexing decimal ciphers before commencing to divide.

3. If there is a remainder after all the figures in the dividend are used, the division may be continued by annexing ciphers, each cipher annexed adding one to the number of decimal places in the dividend. Four or five decimals in the quotient are generally enough.

4. To divide by 10, 100, 1000, etc., remove the decimal point in the dividend as many places to the left as there are ciphers in the divisor (157).

Problems.

(1) $9)72.9($ (2) $31)7.29($ (3) $25).625($ (4) $75)9.375($ (5) $38)467.4($

(6) $3.5)4.725$ miles$($ (7) $2.73)8.8998$ tons$($ (8) $3.28)\$6018.80($

Find the value of—

9. .675 ÷ 5; 81.25 ÷ 25; 82 ÷ 16; .0496 ÷ 18.

10. .025 ÷ 2.5; .0025 ÷ .25; 25 ÷ .25; 25 ÷ .0025.

11. .05 ÷ 100; .007 ÷ 2000; 700 ÷ .0002; 1 ÷ .00001.

12. \$300.75 ÷ 25; \$500 ÷ .625; 1760 yards ÷ .44 yards.

13. Divide one by one hundredth; one hundredth by one thousand; one hundred-thousandth by one hundredth.

14. What is the quotient of two thousand five millionths divided by four hundred one? By 401 hundred-thousandths?

15. If the dividend is 990 and the divisor is 2.475, what is the quotient? If the divisor is .0495?

16. What is the divisor if the quotient is .3243, and the dividend is 3243? If the dividend is 32.43?

17. If 144 bushels of wheat cost \$316.80, what is the price of 1 bushel? If 75 bushels cost \$159.375?

18. How many bins, each holding 100.25 bushels, are needed to contain 2005 bushels of wheat? 802 bushels?

19. How many bottles are needed to hold 21.5 gallons of cologne, if each bottle contains .03125 of a pint?

20. $(500 \times .05) \div .005$; $1 - (\overline{.006 \div 600} \times 1000) = ?$

21. From the quotient of 300 by .003 take their difference.

22. Divide 10 thousandths by 10 thousand, and multiply the product by the difference between 5 tens and 5 tenths.

23. Divide 6 thousand by 6 thousandths, and multiply the quotient by 400 times 25 ten-thousandths.

24. If $8\frac{1}{2}$ yards of silk cost \$18.0625, how much will $13\frac{3}{4}$ yards cost? 20.5 yards? 50.05 yards?

25. In four days a merchant's sales were \$12.75, \$800.50, \$75.37$\frac{1}{2}$, and \$1500.25$\frac{1}{4}$. What were his average daily sales?

UNITED STATES MONEY.

346. United States money is the legal money, or currency, of the United States.

347. Table of United States Money.

10 mills (m.)	are	*1 cent,*	*c.*, or *ct.*
10 cents	"	*1 dime,*	*d.*
10 dimes, or *100 cents*	"	*1 dollar,*	*\$.*

\$1 = 100 c. = 1000 m.

348. The ***Dollar*** is the principal unit of United States money.

Notation and Numeration of United States Money.

349. Dollars are written as integers, with the dollar sign (*\$*) prefixed.

Thus, 5 dollars are written \$5; 25 dollars, \$25.

In business, dimes are regarded as a number of cents, and dimes and cents are expressed together as cents.

350. Since 100 cents are 1 dollar, 1 cent is 1 hundredth of a dollar, and cents are expressed as hundredths of a dollar.

Cents are written: —

1 cent,	*\$.01*	*4 cents,*	*\$.04*	*7 cents,*	*\$.07*
2 cents,	*.02*	*5 cents,*	*.05*	*8 cents,*	*.08*
3 cents,	*.03*	*6 cents,*	*.06*	*9 cents,*	*.09*

351. To express a number of cents as a decimal of a dollar, two figures are needed.

352. When a decimal of a dollar is expressed by two figures, it denotes a number of cents.

1. How many cents are there in \$.03? \$.13? \$.25? \$.08?

353. Since 10 mills are 1 cent, 1 mill is 1 tenth of 1 cent, or 1 thousandth of a dollar.

Mills are written: —

1 mill,	*\$.001*	*4 mills,*	*\$.004*	*7 mills,*	*\$.007*
2 mills,	*.002*	*5 mills,*	*.005*	*8 mills,*	*.008*
3 mills,	*.003*	*6 mills,*	*.006*	*9 mills,*	*.009*

354. To express a number of mills as a decimal of a dollar, three decimal figures are needed.

355. When a decimal of a dollar is expressed by three figures, the first two to the right of the decimal point denote cents, and the third denotes mills.

2. How many cents and how many mills are in the decimal \$.950? \$.731? \$.503? \$.035?

356. Principle.

United States currency is expressed according to the decimal system of notation.

Exercises.

1. Express in figures, or write, *seven dollars twelve cents.*

357. Model. — Write seven dollars as an integer with the dollar sign prefixed, $7; then twelve cents as hundredths of a dollar with the decimal point prefixed, giving $7.12, the expression required.

Express in figures the following sums of United States money: —

Two dollars fourteen cents; eight dollars twenty-five cents; twelve cents; seventy-four dollars seventy-five cents; ninety-eight dollars ten cents; 100 dollars 12 cents 5 mills; 130 dollars 7 cents; 225 dollars 30 cents; 300 dollars 5 cents 3 mills; 450 dollars 62 cents 5 mills; 500 dollars 5 mills; 1000 dollars 1 cent; 2000 dollars 2 mills.

2. Express in words, or numerate and read, *$12.375.*

358. Model. — Read first the integer as dollars, giving $12; then the first two figures to the right of the decimal as cents, giving 37 cents; then the third figure to the right of the decimal as mills, giving 12 dollars 37 cents 5 mills, the expression required.

Express in words the following sums of United States money: —

1. *$12; $15; $18.*	4. *$60.07; $70.05.*	7. *$.007; $.005.*
2. *$21.10; $34.25.*	5. *$80.04; $92.08.*	8. *$2.005; $6.007.*
3. *$47.36; $58.70.*	6. *$1.125; $4.065.*	9. *$3.065; $9.014.*

10. *$100.001; $2051.205; $350.35; $401.004; $561.015.*

11. *$1000.001; $2050.025; $3500.35; $4001.004; $5010.05.*

12. *$.018$\frac{3}{4}$; $10000.50; $3000.007; $50000.50; $100000.005.*

Computations in United States Money.

359. In business calculations, if the mills in the final result are 5 or more than 5, they are regarded as being equal to one cent; if less than 5, they are not regarded at all.

Thus, $12.626 are regarded as $12.63; and $12.621 as $12.62.

360. Principle.

All operations upon United States currency are performed in the same manner as similar operations upon integers and decimals.

Written Exercises.

361. *Example* 1. — Find the sum of \$10.50, \$.37½, \$25.48, \$100.

SOLUTION.

$10.50
.375
25.48
100.

$136.355

EXPLANATION.—Since only like orders of integral units and decimal units can be added, write the parts so that units of the same order shall stand in the same column.

Begin at the right, and add as in decimals and integers (334), and place the decimal point before cents in the sum, giving $136.355, the sum required.

362. *Example* 2. — From \$20.08 subtract \$10.027.

SOLUTION.

$20.08
10.027

$10.053

EXPLANATION.—Since only like orders of integral units and decimal units can be taken one from the other, etc. (337).

Begin at the lowest order, etc., giving $10.053, the difference required.

363. *Example* 3. — Find the product of \$100.45 by 8.7.

SOLUTION.

$100.45
8.7

70315
80360

$873.915

EXPLANATION. — Begin with the lowest order of the multiplier, etc.

Since hundredths multiplied by tenths produce thousandths, the product must contain three decimal places, or as many decimal places as are in both factors, giving $873.915, the product required.

364. *Example* 4. — How much is \$302.375 ÷ \$41?

SOLUTION.

$41)302.375(7.375
287

153
123

307
287

205
205

EXPLANATION. — Begin with the highest order of the dividend, etc.

Since the dividend contains three decimal places, and the divisor has none, the quotient must have three decimal places, giving $7.375, the quotient required.

365. Rule for Computations in United States Money.

Write the numbers, and add, subtract, multiply, and divide as in integers and decimals, and place the decimal point as in decimals.

Problems.

Add together the following sums of money: —

(1)	(2)	(3)	(4)	(5)
$7.35	$12.075	$100.05	$75.50	$407.381
8.50	.625	1.005	.75	70.65
7.75	8.50	.07	4.007	5.006
8.62	.375	95.736	100.	.76
4.25	40.	140.068	.568	.078

6. $75.07 + $9.075 + $100 + $76.075 + $9000.37.

7. $90.07½ + $400.31 + $75.06 + $80 + $73.18¾.

8. What is the amount of $76.25, $800, $78.25, $98.625?

9. Add together $50, $400.3125, $8, $40.50, and $78.37½.

10. A merchant has in one bank $3500, in another $4306.50, and in a third $10050.75; and has on hand $275.37½. How much money has he in all?

Find the remainder of—

(11)	(12)	(13)	(14)	(15)
$375.50	$8.70	$.5625	$100.	$75.25
9.75	.378	.375	7.625	9.1875

16. $8.50 — $1.75; $76.25 — $1.625; $90 — $.3125.

17. $1 — $.875; $50.50 — $.505; $1000 — $76.25¾.

18. What is the difference between $400 and $376.375?

19. From $200 take $100.25; from $100 take $1.001.

20. An agent sold a farm for $10000, which was $250.625 more than he paid for it. What was its cost?

Find the product in each of the following: —

(21)	(22)	(23)	(24)	(25)	(26)
$150	$8.37½	$25.705	$100.005	$9.08	$700.50
7½	8	9.5	10.5	3.2¼	3.25

27. $.625 × 15; $10.005 × 25; $75.37½ × 15.

28. $100 × 7.5; $250.25 × .37½; $1200.25 × 6.25¼.

29. Find the product of \$375.625 multiplied by $31\frac{1}{4}$.

30. How many dollars are 37.5 times \$40.37$\frac{1}{2}$?

31. How much would 33 yards of carpeting cost at \$1.37$\frac{1}{2}$? At \$1.56$\frac{1}{4}$? At \$1.66$\frac{2}{3}$? At \$2.12$\frac{1}{2}$?

Find the quotient of—

(32) *12)\$.625* (33) *2.5)\$100.50* (34) *.075)\$20.50* (35) *2.25)\$730*

36. \$7.50 ÷ 5; \$70.50 ÷ 5.5; \$700.50 ÷ .05.

37. \$9.87$\frac{1}{2}$ ÷ 8; \$17.56$\frac{1}{4}$ ÷ \$15; \$500.375 ÷ 6.25.

38. Find the quotient of \$30.5625 divided by 75.

39. How many times are \$2.75 contained in \$20.5025?

40. If you save 62$\frac{1}{2}$ cents a day, how long would it take to save \$200.12$\frac{1}{2}$? To save \$500? \$62.50? \$750?

41. Divide \$4 into 8 equal parts; 50 equal parts; 150.

42. How many bushels of wheat, at a freight of 3$\frac{1}{2}$ cents, can a dealer have transported for \$65.625?

43. If a housekeeper paid \$.75 for a piece of beef-steak at \$.18$\frac{3}{4}$ a pound, how much did it weigh?

BILLS.

366. A ***Bill of Goods*** is a written statement of goods sold.

367. A ***Debtor*** is one who owes money, goods, or services to another.

368. A ***Creditor*** is one to whom money, goods, or services are owed by another.

369. The ***Footing*** of a bill is the entire cost of the items or articles.

The character @ signifies *at*. Thus, 6 yards of muslin @ \$.12$\frac{1}{2}$, means 6 yards of muslin at 12$\frac{1}{2}$ cents per yard.

1. Bill not Receipted.

PHILADELPHIA, *July 1, 1901.*

Mr. William Hunter,

Bought of HUDSON BROS.

12 lb. Hyson Tea,	@	$\$1.12\frac{1}{2}$	
5 " Java Coffee,	@	$.37\frac{1}{2}$	
10 " Crushed Sugar,	@	$.14$	

2. Bill Receipted.

NEW YORK, *July 30, 1901.*

Mr. Henry Fenton,

Bought of STUART & CO.

15 yd. Sheeting,	@	$\$.18\frac{3}{4}$	
10 " Muslin,	@	$.16\frac{2}{3}$	
$8\frac{3}{8}$ *" Cambric,*	@	$.25$	

Received Payment,

Stuart & Co.

3. Boston, March 30th, 1901. John Williams bought of Edgar Peirson, 9 yards of calico @ $\$.12\frac{1}{2}$; 15 yards cambric @ \$.25; 16 yards sheeting @ $\$.18\frac{3}{4}$; 5 dozen towels @ \$3.75.

4. April 13, 1901, New York. J. Williams & Co. sold to James Garsed, 12 yards cassimere @ $\$1.12\frac{1}{2}$; 20 yards doeskin @ $\$.91\frac{1}{4}$; 18 yards flannel @ $\$.62\frac{1}{2}$; 10 yards alpaca @ $\$.87\frac{1}{2}$. Make out and receipt the bill.

5. William Watson bought of Henry Stout, Richmond, May 1, 1901. 12 pounds Oolong tea @ \$1.10; 8 pounds Young Hyson tea @ \$.95; 15 pounds Rio coffee @ $\$.37\frac{1}{2}$; 20 pounds sugar @ $\$.12\frac{1}{4}$. Make out and receipt the bill.

6. Charles Foulke bought of T. P. Smith & Bro., Philadelphia, Nov. 15, 1901 8 pair calf boots, @ $\$5.62\frac{1}{2}$; 10 pair brogans, @ $\$1.12\frac{1}{2}$; 12 pair ladies' kid boots, @ $\$1.66\frac{2}{3}$; 15 pair boys' gaiters, @ \$2.25. Make out and receipt the bill.

SECTION XIX.

REDUCTION OF DECIMALS.

CASE I.

To Change a Decimal to a Common Fraction.

1. How is .2 expressed as a decimal fraction? .3? .5? .7?
2. What decimal fraction of a dollar is \$.5? \$.25? \$.75?
3. Express .5 as a decimal fraction, and change it to halves.
4. How many halves in $\frac{50}{100}$? $\frac{500}{1000}$? In .5? .50? .500?
5. How many fourths in $\frac{25}{100}$? $\frac{75}{100}$? In .50? In .250?
6. In $\frac{4}{10}$ are how many fifths? $\frac{20}{100}$? In .4? .6? .80?
7. In $\frac{50}{100}$, how many twentieths? Tenths? Fifths? Halves?
8. Express .25 as a common fraction in its lowest terms; .375 of an acre; .625 of a ton; .0625 of a foot.

370. Principle.

A decimal fraction is a decimal with the denominator expressed.

Written Exercises.

371. *Example* 1. — Change .875 to an equivalent common fraction in its lowest terms.

SOLUTION.

$.875 = \frac{875 \div 125}{1000 \div 125} = \frac{7}{8}$

875)1000(1
875
125)875(7
875

EXPLANATION.—Since .875 is 875 thousandths, it may be expressed in the form of a common fraction by writing 875 as the numerator and 1000 as the denominator, giving $\frac{875}{1000}$. Dividing both terms by 125, their greatest common divisor, gives $\frac{7}{8}$, the common fraction required.

372. *Example* 2. — Express $2.37\frac{1}{2}$ as a mixed number.

SOLUTION.

$2.37\frac{1}{2} = 2\frac{37\frac{1}{2}}{100} = 2\frac{3}{8}$

$\frac{37\frac{1}{2}}{100} = \frac{75}{2} \times \frac{1}{200} = \frac{3}{8}$

EXPLANATION. — Since $.37\frac{1}{2}$ is $37\frac{1}{2}$ hundredths, it may be written $\frac{37\frac{1}{2}}{100}$.

The complex fraction changed to a simple fraction equals $\frac{3}{8}$; and $\frac{3}{8}$ united with 2 gives $2\frac{3}{8}$, the mixed number required.

373. Rule to Change a Decimal to a Common Fraction.

For the numerator, write the decimal with the decimal point omitted; and for the denominator, write 1 *with as many ciphers annexed as there were places in the given decimal. Reduce the common fraction thus formed to its lowest terms.*

Problems.

Change, or reduce, to fractions in their lowest terms —

1. .4; .6; .8; .9.
2. .25; .45; .75.
3. .625; .875.
4. .825; .0025.
5. \$.75; \$.80; \$.125.
6. \$.008; \$.0065.

7. Reduce .0075 and .0004 to equivalent common fractions.

8. Change .00375 to a common fraction in its lowest terms.

Change to common fractions in their lowest terms —

9. .12½; .37½.
10. .62½; .87½.
11. .16⅔; .33⅓.
12. .06¼; .18¾.
13. .22⅔; .44⅔.
14. .66⅔; .31¼.
15. \$.87½; \$.56¼.
16. \$.41⅔; \$.91¾.
17. \$.004; \$.002½.

Express as mixed numbers —

18. 3.25; 4.50; 5.75.
19. 1.625; 5.085; 7.005.
20. 2.06¼; 4.12½; 6.33⅓.
21. \$4.50; \$6.65; \$8.80.
22. \$3.06; \$5.08; \$7.015.
23. \$6.085; \$7.66⅔; \$8.00¾.

CASE II.

To Change a Common Fraction to a Decimal.

1. In 1 dollar are how many tenths of a dollar? In \$2?
2. In 1 are how many tenths? In ½ how many?
3. How many tenths in 1 divided by 2? 1 ÷ 5?
4. How many tenths in 2 divided by 5, or ⅖?

374. Analysis.—Since 1 is 10 tenths, 2 is 2 times 10 tenths, or 20 tenths; 20 tenths divided by 5 are 4 tenths. Hence, $\frac{2}{5} = \frac{4}{10}$, or .4.

5. How many hundredths in 1? In 1 ÷ 2, or ½?
6. In ⅕ how many tenths? Hundredths? Thousandths?
7. How many hundredths in ⅖? Thousandths in ⅗?

8. In $\frac{3}{4}$ how many hundredths? Thousandths? Ten-thousandths?

375. Principle.

The value of a fraction is the quotient of the numerator divided by the denominator.

Written Exercises.

376. *Example* 1. — Change $\frac{3}{4}$ to an equivalent decimal.

SOLUTION.

$\frac{3}{4} = .75.$

$4)3.00$

$.75$

EXPLANATION. — $\frac{3}{4}$ is $3 \div 4$, or $\frac{1}{4}$ of 3.

Since 1 is 10 tenths, 3 is 3 times 10 tenths, or 30 tenths; $\frac{1}{4}$ of 30 tenths is 7 tenths, with 2 tenths remaining.

Since 1 tenth is 10 hundredths, 2 tenths are 2 times 10 hundredths, or 20 hundredths; $\frac{1}{4}$ of 20 hundredths is 5 hundredths.

The result is 7 tenths 5 hundredths, or *.75*, the decimal required.

377. *Example* 2. — Change $6\frac{1}{4}$ to a mixed decimal.

SOLUTION.

$6\frac{1}{4} = 6 + \frac{1}{4} = 6.25.$

$4)1.00$

$.25$

EXPLANATION. — $6\frac{1}{4}$ equals $6 + \frac{1}{4}$; and $\frac{1}{4}$ is $1 \div 4$, or $\frac{1}{4}$ of 1.

Since 1 is 10 tenths, $\frac{1}{4}$ of 10 tenths is 2 tenths, etc. (Ex. 1). The result is $6 + .25$, or *6.25*, the mixed decimal required.

378. Rule to Change a Common Fraction to a Decimal.

Annex ciphers to the numerator, and divide by the denominator; and from the right hand of the quotient point off as many places for decimals as there were decimal ciphers annexed. The quotient will be the decimal required.

Problems.

Change to equivalent decimals —

1. $\frac{2}{5}$; $\frac{3}{4}$; $\frac{7}{8}$.
2. $\frac{7}{1\,2}$; $\frac{9}{2\,0}$; $\frac{1\,1}{2\,5}$.
3. $\frac{2\,1}{4\,0}$; $\frac{3\,2}{5\,0}$; $\frac{4\,3}{7\,5}$.
4. $\frac{1\,5}{1\,8}$; $\frac{4\,3}{5\,0}$; $\frac{1\,2\,5}{1\,0\,0}$.
5. $\frac{3}{4\,0}$; $\frac{3}{8\,0}$; $\frac{3}{1\,6\,0}$.
6. $\frac{4}{3\,7\,5}$; $\frac{6}{6\,2\,5}$; $\frac{8}{8\,7\,5}$.
7. $\$\frac{3}{8}$; $\$\frac{7}{1\,0}$; $\$\frac{1\,1}{1\,2}$.
8. $\$\frac{2\,5}{1\,0\,0}$; $\$\frac{7}{1\,6}$; $\$\frac{7\,5}{1\,0\,0}$.
9. $\$\frac{6\,2\,5}{1\,0\,0\,0}$; $\frac{4}{5}$ of $\$9\frac{4}{5\,0}$.
10. Change $\frac{3}{7}$, $\frac{8}{9}$, $\frac{1\,0}{1\,1}$ to decimals of five places each.
11. Reduce to decimals of five orders each $\frac{9}{3\,2}$, $\frac{4\,3}{4\,8}$, $\frac{6\,1}{6\,4}$.

12. $7\frac{1}{8}$; $8\frac{5}{10}$.	15. $10.02\frac{1}{5}$; $12.04\frac{3}{5}$.	18. $\$7\frac{3}{5}$; $\$8\frac{3}{4}$.
13. $4\frac{7}{10}$; $7\frac{17}{20}$.	16. $15.068\frac{1}{8}$; $20.08\frac{7}{8}$.	19. $\$.6\frac{1}{4}$; $\$.01\frac{1}{8}$.
14. $5.2\frac{1}{2}$; $9.3\frac{1}{4}$.	17. $25.00\frac{1}{2}$; $50.00\frac{1}{4}$.	20. $\$1.00\frac{1}{3}$; $\$4\frac{1}{8}$.

Review.

1. What orders of decimal units only can be added? State the principle of addition of decimals. Repeat the rule. How many decimal places in the sum of tenths and tenths? Tenths and hundredths? Tenths and thousandths? Hundredths and thousandths?

2. When only can one decimal fraction be taken from another? One decimal from another? One order of decimal units from another? State the principle of subtraction of decimals. Repeat the rule. How many decimal places in the difference between tenths and tenths? Tenths and hundredths? Hundredths and thousandths?

3. State the principle of multiplication of decimals. How many decimal places in the product of tenths by tenths? Tenths by hundredths? Hundredths by hundredths? Thousandths by hundredths? Thousandths by thousandths? In the product of any two decimals? Repeat the rule. What is to be done when the number of decimal places in the product does not equal the number in both factors? How do you multiply a decimal by 10, 100, 1000, etc.?

4. Of what two factors is the dividend the product? In decimals, the dividend contains as many decimal places as what two factors? Of what order of units is the quotient of tenths divided by tenths? Hundredths by hundredths? Hundredths by tenths? Thousandths by tenths? Of any decimal divided by another decimal? State the principle of division of decimals. Repeat the rule. What is to be done when the number of decimal places in the quotient is too small? If the dividend has not so many places as the divisor before division? If there is a remainder? How do you divide a decimal by 10, 100, etc.?

5. What is ***United States Money?*** What is the principal unit? How are dimes usually regarded? What decimal of a dollar is one cent? How many figures are needed to express cents decimally? To express mills? State the principle of notation. The general principles of all computations in United States money. Define ***Bill of Goods.*** Define debtor. Creditor. Footing of a bill.

6. In what two ways may a decimal be expressed? State the principle of changing a decimal to a common fraction. Repeat the rule. State the principle of changing a common fraction to a decimal. Repeat the rule.

SECTION XX.

DENOMINATE NUMBERS.

379. Denomination is the name of the unit of a concrete number.

Thus, *dollar* is the denomination of *3 dollars;* gallon, of 5 gallons, etc.

380. A ***Denominate Unit*** is *one* of any denomination.

Thus, *1 yard* is a denominate unit; so, also, is 1 ton, 1 mile, etc.

381. A ***Denominate Number*** is one or more units of any denomination.

Thus, *1 foot* is a denominate number; so, also, is 2 quarts, etc.

382. *Denominate Numbers* are either *Simple* or *Compound.*

383. A ***Simple Denominate Number*** is a number expressed in units of only one denomination.

Thus, *5 yards* is a simple denominate number; so, also, is 2 feet.

384. A ***Compound Denominate Number*** is a number expressed in units of two or more denominations of the same nature.

Thus, *5 yards 2 feet* is a compound number; also, 4 gallons 2 quarts.

385. *Compound Denominate Numbers* are generally called *Compound Numbers.*

MEASURES.

386. Measures may be divided into *six classes:* —

1. *Measures of Extension.*
2. *Measures of Capacity.*
3. *Measures of Weight.*
4. *Measures of Money.*
5. *Measures of Circles.*
6. *Measures of Time.*

387. MEASURES OF EXTENSION.

388. Extension may be a line, a surface, or a solid.

A ***Line*** is that which has length only.

A ***Surface*** is that which has length and breadth.

A ***Solid,*** or ***Body,*** is that which has length, breadth, and thickness.

389. Measures of extension are of three classes: *measures of lines, measures of surfaces,* and *measures of solids.*

I. Linear Measures.

A *line* has but one dimension, — *length.*

Linear Measures are used in measuring lines and distances.

The units or denominations of linear measures are *inch, foot, yard, rod,* and *mile.*

Table.

12 inches (in.)	are	*1 foot, ft.*	1 *inch.*
3 feet	"	*1 yard, yd.*	
5 $\frac{1}{2}$ yards, or *16 $\frac{1}{2}$ feet*	"	*1 rod, rd.*	
320 rods	"	*1 mile, mi.*	

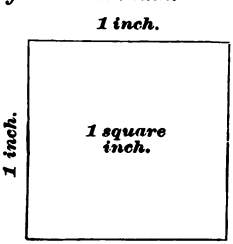
1 mile = 320 rd. = 1760 yd. = 5280 ft.

In measuring *cloth,* or goods sold by the yard, the linear yard is divided into *halves, quarters, eighths,* and *sixteenths.*

II. Surface Measures.

A *surface* has two dimensions, — *length* and *breadth.*

390. A ***Square*** is a surface having four equal straight sides, and four equal corners or angles.



A ***Square Inch*** is a surface 1 inch long and 1 inch wide.

A ***Square Foot*** is a surface 1 foot long and 1 foot wide.

Surface Measures, or ***Square Measures,*** are used in ascertaining the extent of surfaces; as of boards, plastering, land, etc.

The units or denominations of surface or square measures are *square inch, square foot, square yard, square rod, acre,* and *square mile.*

Table.

144	*square inches (sq. in.)*	are	*1 square foot, sq. ft.*
9	*square feet*	"	*1 square yard, sq. yd.*
$30\frac{1}{4}$	*square yards*	"	*1 square rod, sq. rd.*
160	*square rods*	"	*1 acre, A.*
640	*acres*	"	*1 square mile, sq. mi.*

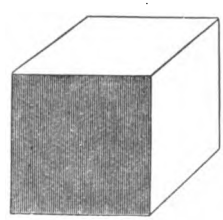
1 acre = 160 sq. rd. = 4840 sq. yd. = 43560 sq. ft.

In measuring land, 16 square rods are 1 square chain (*sq. ch.*), and 10 square chains are 1 acre.

III. Solid Measures.

A *solid* has three dimensions, — *length, breadth,* and *thickness.*

391. A ***Cube*** is a solid having six equal square sides, called faces.



A Cube.

A ***Cubic Inch*** is a solid 1 inch long, 1 inch wide, and 1 inch thick.

A ***Cubic Foot*** is a solid 1 foot long, 1 foot wide, and 1 foot thick.

Solid or ***Cubic Measures*** are used in ascertaining the contents or volumes of solids; such as *boxes, timber, rooms,* etc.

The units or denominations of solid or cubic measures are *cubic inch, cubic foot,* and *cubic yard.*

Table.

1728	*cubic inches (cu. in.)*	are	*1 cubic foot, cu. ft.*
27	*cubic feet*	"	*1 cubic yard, cu. yd.*

1 cubic yd. = 27 cu. ft. = 46656 cu. in.

In measuring wood, 128 cubic feet are 1 cord (*cd.*).

392. MEASURES OF CAPACITY.

393. The measures of capacity are of two classes: *measures of liquids* and *measures of dry substances.*

Note. — The measures of capacity are all cubic measures; each unit of capacity representing a certain number of cubic inches.

I. LIQUID MEASURES.

Liquid Measures are used in measuring liquids; such as *water, milk, oil, liquors,* etc.

The units or denominations of liquid measure are *gill, pint, quart,* and *gallon.*

Table.

4 gills (gi.)	are	*1 pint, pt.*
2 pints	"	*1 quart, qt.*
4 quarts	"	*1 gallon, gal.*

1 gal. = 4 qt. = 8 pt. = 32 gi.

In measuring the capacity of cisterns, etc., 31½ gallons are 1 barrel (*bbl.*); 63 gallons are 1 hogshead (*hhd.*).

The gallon, liquid measure, contains 231 cu. in.

II. DRY MEASURES.

Dry Measures are used in measuring dry substances; such as *grain, fruit, vegetables, lime, coal,* etc.

The units or denominations of dry measure are *pint, quart, peck,* and *bushel.*

Table.

2 pints (pt.)	are	*1 quart, qt.*
8 quarts	"	*1 peck, pk.*
4 pecks	"	*1 bushel, bu.*

1 bu. = 4 pk. = 32 qt. = 64 pt.

4 quarts, or 1 half-peck, or 1 gallon, dry measure, contains 268⅘ cubic inches. 1 bushel contains 2150.42 cubic inches.

394. MEASURES OF WEIGHT.

395. The measures of weight are of two classes: *Troy weights* and *avoirdupois weights.*

I. TROY WEIGHTS.

Troy Weights are used in weighing gold, silver, etc.

The units or denominations of Troy weights are *grain, pennyweight, ounce,* and *pound.*

Table.

24 grains (gr.)	are	*1 pennyweight,*	*pwt.*
20 pennyweights	"	*1 ounce,*	*oz.*
12 ounces	"	*1 pound,*	*lb.*

1 lb. = 12 oz. = 240 pwt. = 5760 gr.

II. AVOIRDUPOIS WEIGHTS.

Avoirdupois Weights are used in weighing all coarse and heavy articles; such as *groceries, iron, coal,* etc.

The units or denominations of avoirdupois weights are *ounce, pound, hundred-weight,* and *ton.*

Table.

16 ounces (oz.)	are	*1 pound,*	*lb.*
100 pounds	"	*1 hundred-weight,*	*cwt.*
20 hundred-weight	"	*1 ton,*	*T.*

1 T. = 20 cwt. = 2000 lb. = 32000 oz.

In weighing coal and iron, and in a few other cases, the gross hundred-weight of 112 pounds, and the gross ton of 2240 pounds are used.

The pound avoirdupois weighs 7000 grains Troy.

396. MEASURES OF MONEY.

397. ***Money*** is of two kinds: *coin* and *paper money.*

Coin is metal stamped to be used as money.

Paper Money is a substitute for coin, and consists of notes issued by the government and by banks.

398. Currency is either coin or paper substitute for money.

I. United States Money.

United States Money is the legal currency of the United States.

The units or denominations of United States money are *mill, cent, dime, dollar,* and *eagle.*

Table.

10 mills (m.)	are	*1 cent,*	*ct. or c.*
10 cents	"	*1 dime,*	*d.*
10 dimes	"	*1 dollar,*	*$.*
10 dollars	"	*1 eagle,*	*e.*

$1 = 10 d. = 100 ct. = 1000 m.

The ***Coins*** of the United States are of gold, silver, nickel, and bronze.

The ***Gold Coins*** are the double-eagle, eagle, half-eagle, and the quarter-eagle.

The ***Silver Coins*** are the dollar, half-dollar, quarter-dollar, and dime.

The ***Nickel Coin*** is the five-cent piece.

The ***Bronze Coin*** is the cent.

II. English Money.

English Money is the legal currency of Great Britain.

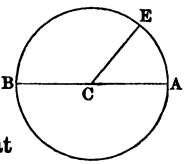
The units or denominations of English money are *farthing, penny, shilling,* and *pound.*

Table.

4 farthings (far. or qr.)	are	*1 penny,*	*d.*
12 pence	"	*1 shilling,*	*s.*
20 shillings	"	*1 pound,*	*£.*

1 £ = 20 s. = 240 d. = 960 qr.

399. MEASURES OF THE CIRCLE.



400. A ***Circle*** is a plane or flat surface bounded by a curved line, which is everywhere at an equal distance from a point within called the centre.

A ***Circumference*** is the line that bounds a circle.

401. A ***Degree*** is one of the 360 equal parts into which the circumference of every circle is divided.

402. An ***Angle*** is the opening between two lines that meet at the same point.

Thus, the lines E C and A C, which meet at C, form the angle A C E.

Circular or ***Angular Measures*** are used to measure angles, to reckon latitude, longitude, time, etc.

The units or denominations of circular or angular measures are *second, minute, degree,* and *circumference.*

Table.

60 seconds (″)	are	*1 minute,*	′
60 minutes	"	*1 degree,*	°
360 degrees	"	*1 circumference,*	*C.*

1 C. = 360° = 21600′ = 1296000″.

403. MEASURES OF TIME.

Measures of Time are used in measuring time.

The units or denominations of time are *second, minute, hour, day,* and *year.*

Table.

60 seconds (sec.)	are	*1 minute,*	*min.*
60 minutes	"	*1 hour,*	*hr.*
24 hours	"	*1 day,*	*da.*
365 days	"	*1 common year,*	*yr.*
366 days	"	*1 leap year,*	*yr.*

1 yr. = 365 da. = 8760 hr.= 525600 min.

Also, 7 days are 1 week (*wk.*); 12 months (*mo.*) are 1 year, and 100 years are 1 century (*C.*).

In most business transactions, 30 days are considered as a month, and 12 months as a year.

The names of the months and the number of days in each month are as follows: —

		Days.			Days.
January,	*Jan.*,	31.	July,	*July*,	31.
February,	*Feb.*,	28 or 29.	August,	*Aug.*,	31.
March,	*Mar.*,	31.	September,	*Sept.*,	30.
April,	*Apr.*,	30.	October,	*Oct.*,	31.
May,	*May*,	31.	November,	*Nov.*,	30.
June,	*June*,	30.	December,	*Dec.*,	31.

404. In ***counting*** certain articles, the units or denominations *dozen*, *gross*, and *great gross* are used.

Table.

12 things	are	*1 dozen*,	*doz.*
12 dozen	"	*1 gross*,	*gro.*
12 gross	"	*1 great gross*,	*grt. gro.*

1 grt. gro. = 12 gro. = 144 doz. = 1728 things.

405. In ***buying and selling paper***, the units or denominations *sheet*, *quire*, *ream*, *bundle*, and *bale* are used.

Table.

24 sheets	are	*1 quire.*	*2 reams*	are	*1 bundle.*
20 quires	"	*1 ream.*	*5 bundles*	"	*1 bale.*

Notation and Numeration of Compound Numbers.

406. In integers and in decimals, the orders of units increase and decrease by the uniform rate or scale of 10.

In compound numbers there is no uniform scale of increase and decrease.

407. The different orders of units of compound numbers increase, as do integers and decimals, from right to left, the higher orders, or denominations, being always written at the left, and the lower orders, or denominations, at the right.

Exercises.

Express, or write, as a denominate number with proper abbreviations, *sixteen miles four rods two yards six inches.*

408. Model. — Sixteen miles four rods two yards six inches is written *16 mi. 4 rd. 2 yd. 6 in.*, the expression required.

Express in words, or read, *3 A. 14 sq. rd. 3 sq. yd. 8 sq. ft.*

409. Model. — 3 A. 14 sq. rd. 3 sq. yd. 8 sq. ft. is read, *three acres fourteen square rods three square yards eight square feet.*

1. Express as denominate or compound numbers: —

Nine yards; twenty cords; eight square yards; nine cubic yards sixty cubic inches; nine yards two feet six inches; seven square yards ninety square inches; five miles two rods two yards two feet five inches; seven acres ten square rods twenty square feet.

Eight bushels three pecks; sixty gallons two quarts; three pecks five quarts one pint; two hogsheads ten gallons; forty bushels two pecks six quarts; thirty barrels two quarts one pint two gills.

Ten pounds six ounces; six tons eight hundred-weight; three ounces ten pennyweights twenty grains; one hundred tons ninety pounds four ounces. Three dollars ten cents; ten pounds four shillings ten pence; twenty-five dollars fifty cents five mills; one hundred pounds ten shillings six pence three farthings.

Ten hours thirty minutes; three hundred days; ninety degrees thirty minutes; fifty years seven months; forty minutes twenty seconds; twenty days ten hours eight minutes eighteen seconds.

2. Express in words, or read, the following compound numbers: —

8 ft. 7 in.; 12 A. 8 sq. ch.; $7\frac{3}{4}$ yd.; 12 cd. 75 cu. ft.; 10 sq. rd. 9 sq. yd. 6 sq. ft.; 8 rd. 30 yd. 2 ft. $6\frac{1}{2}$ in.; 100.25 sq. ch.; 7 cu. yd. 8 cu. ft. 100.5 cu. in.; 180 A. 100 sq. rd. 25 sq. yd.; 1500 cd. $90\frac{7}{8}$ cu. ft.

4 qt. $2\frac{1}{2}$ pt.; 10 bu. 8 pk.; 300 gal. 2 qt. 1 pt. $3\frac{1}{4}$ gi.; 102 bbl. 22.75 gal.; 40 bu. 3 pk. 2 qt.; 400.5 bu.; 275 hhd. 52 gal. 2 qt. 1 pt.; 200 bu. 3 pk. 2 qt. 1.05 pt.; 3000 hhd. 50 gal.; 125 gal. 2 qt. 1 pt.

3 oz. 10 pwt. 15 gr.; 15 T. 15 cwt. 90.75 lb.; 5 lb. 8 pwt. 18 gr.; 75.075 tons; 100 T. 50 lb. $10\frac{7}{16}$ oz.; 125 lb. 10 oz. 15 pwt. 20 gr.

$5.25; £13 10 s. 6¾ d.; $25.07¾; £50 15 s. 5 d. 3 qr.; $50.005⅜; 12 s. 6.75 d.; $.075; 4 d. 3¾ qr.; $.005.

8 yr. 4 mo.; 75° 15′; 4 wk. 4 da. 10 hr.; 50½°; 24.75 hr.; 7 min. 30 sec.; 365.da. 5 hr. 48 min. 49.7 sec.; 60° 15′ 30″; 40.5 min.

Review.

1. Define ***Denomination.*** What is a denominate unit? A denominate number? Name the two classes of denominate numbers. What is a simple denominate number? A compound denominate number? What are compound numbers?

2. Name six classes of ***Measures.*** Three kinds of ***extension.*** Define line, surface, and solid. Name three classes of measures of extension.

3. For what are ***Linear Measures*** used? Name the units of linear measures. Repeat the table. What dimensions has a surface? Define ***Square.*** What is a square inch? A square foot? For what are surface or square measures used? What are the denominations of surface measures? Repeat the table. Define ***Solid.*** What is a cube? A cubic inch? A cubic foot? For what are solid or cubic measures used? What are the denominations? Repeat the table.

4. Name two classes of measures of ***Capacity.*** For what are liquid measures used? Name the denominations of liquid measures. Repeat the table. For what are dry measures used? Name the denominations. Repeat the table.

5. Name two classes of measures of ***Weight.*** For what are Troy weights used? What are the denominations of Troy weights? Repeat the table. For what are avoirdupois weights used? Name the denominations, and repeat the table. How many grains are one Troy pound? One avoirdupois pound?

6. Name two kinds of ***Money.*** What is coin? Paper money? Currency? What is United States money? Name the denominations of United States money, and repeat the table. Name the coins. What is English money? Repeat the table.

7. What is a ***Circle?*** A circumference? A degree? An angle? For what are measures of circles used? Name the denominations of circular measures, and repeat the table.

8. For what are measures of ***Time*** used? Name the denomina-

tions of time, and repeat the table. Name the months of the year, and tell how many days are in each.

9. In ***Counting***, what units or denominations are used? Repeat the table. In buying and selling paper? Repeat the table.

10. In integers and in decimals, how do the orders of units increase and decrease? In compound numbers, is there any regular rate or scale of increase and decrease? How do the different orders of units in integers, decimals, and compound numbers increase? Where are the highest orders, or denominations, placed? The lowest?

SECTION XXI.

REDUCTION OF DENOMINATE NUMBERS.

410. Reduction of Denominate Numbers is the process of changing them from one denomination to another without changing their value.

411. Reduction of denominate numbers is of two kinds: *Reduction Descending* and *Reduction Ascending.*

412. Reduction Descending is the process of changing a denominate number to an equivalent number expressed in lower denominations.

413. Reduction Ascending is the process of changing a denominate number to an equivalent number expressed in higher denominations.

CASE I.

Reduction Descending.

1. How many inches in 1 ft.? In 2 ft.? In 3 ft.?

2. Change 2 ft. 6 in. to inches.

414. Analysis. — Since 1 foot is 12 inches, 2 feet are 2 times 12 inches, or 24 inches; 24 inches and 6 inches are 30 inches.

3. In 2 gal. are how many quarts? How many pints?

4. Express 3 bu. 2 pk. as pecks; as pints. 5 bu. 3 pk.

5. What is the value of 3 qt. 1 pt. of milk at 8 cents a pint? Of 4 gal. 2 qt.? Of 6 gal. 2 qt. 1 pt.?

6. At 50 cents a foot, what will 1 rd. of wire fence cost? At $.75 a foot, how much will 2 rd. 1 yd. of fence cost?

7. Change $\frac{1}{2}$ lb. avoirdupois to ounces. $4\frac{1}{2}$ lb. $8\frac{1}{4}$ lb.

8. At 10 cents a quart, what will $\frac{1}{2}$ bu. of chestnuts cost?

415. Principle.

A denominate number is changed to lower denominations by multiplication.

Written Exercises.

416. *Example.* — Change 2 mi. 50 rd. 3 yd. to yards.

SOLUTION.

2 mi. 50 rd. 3 yd.
320
690 rd. $\times \frac{1}{2} = \frac{690}{2} = 345$
$5\frac{1}{2}$
3450 yd.
345
3798 yd.

EXPLANATION.—Since 1 mile is 320 rods, 2 miles are 2 times 320 rods, or 640 rods; 640 rods and 50 rods are 690 rods.

Since 1 rod is $5\frac{1}{2}$ yards, 690 rods are 690 times $5\frac{1}{2}$ yards, or 3450 yards plus 345 yards, or 3795 yards; 3795 yards and 3 yards are 3798 yards.

Hence, 2 mi. 50 rd. 3 yd. are *3798* yds.

417. Rule for Reduction Descending.

I. *Multiply the number of the highest denomination given by the number of units of the next lower denomination which equals one of that higher; and to the product add the given number of the lower denomination, if any.*

II. *In the same manner reduce this result to the next lower denomination; and so proceed until the given number is changed to the denomination required.*

Problems.

Change, or reduce —

1. 8 rods to inches; 1 mi. 36 rd. 2 ft. to feet.

2. 2 mi. 7 yd. $1\frac{1}{4}$ in. to inches; 5 sq. yd. to square inches.

3. 2 A. to sq. ft.; 40 sq. rd. 8 sq. ft. $100\frac{1}{2}$ sq. in. to sq. in.
4. 1 cd. to cubic inches; 50 cu. yd. $20\frac{1}{2}$ cu. ft. to cu. in.
5. 100 gal. to pints; $4\frac{1}{2}$ bbl. to quarts; 3.5 hhd. to gills.
6. 20 gals. 3 pt. $2\frac{3}{4}$ gi. to gills; 1 lb. Troy to grains.
7. 1 T. to pounds; 10 T. 5 cwt. 75 lb. $12\frac{3}{8}$ oz. to ounces.
8. \$10 to mills; \$1.07 to cents; \$10.005 to mills.
9. £1 to farthings; £50 10 s. $8\frac{1}{4}$ d. to pence; to far.
10. 1° to seconds; 75° 20′ $35\frac{1}{2}$″ to seconds.
11. 2 da. to seconds; 10 da. 20 hr. 10.75 min. to seconds.
12. 365 da. 5 hr. 48 min. 49.7 seconds to seconds.
13. $\frac{3}{5}$ of a gallon to gills.

SOLUTION.

$\frac{3}{5}$ *gal.* $= \frac{3}{5} \times 4$ *qt.* $= \frac{12}{5}$ *qt.*

$\frac{12}{5}$ *qt.* $= \frac{12}{5} \times 2$ *pt.* $= \frac{24}{5}$ *pt.*

$\frac{24}{5}$ *pt.* $= \frac{24}{5} \times 4$ *gi.* $= \frac{96}{5}$ *gi.*

$\frac{96}{5}$ *gi.* $= 19\frac{1}{5}$ *gills.*

EXPLANATION.—Since 1 gallon is 4 quarts, $\frac{3}{5}$ of a gallon are $\frac{3}{5}$ of 4 quarts, or $\frac{12}{5}$ of a quart.

Since 1 quart is 2 pints, $\frac{12}{5}$ of a quart are $\frac{12}{5}$ of 2 pints, or $\frac{24}{5}$ of a pint.

Since 1 pint is 4 gills, $\frac{24}{5}$ of a pint are $\frac{24}{5}$ of 4 gills, or $\frac{96}{5}$ of a gill, which equals $19\frac{1}{5}$ gills.

Change, or reduce —

14. $\frac{3}{8}$ of a mi. to yd.; $\frac{3}{5}$ rd. to ft.; $\frac{3}{10}$ yd. to in.; $4\frac{2}{7}$ ft. to in.
15. $\frac{3}{5}$ of a sq. yd. to sq. in.; $\frac{3}{7}$ A. to sq. yds.; $\frac{4}{9}$ sq. ch. to sq. ft.
16. $\frac{2}{7}$ of a cd. to cu. ft.; $\frac{3}{5}$ hhd. to qt.; $\frac{1}{100}$ gal. to gi.
17. $\frac{2}{9}$ of a bu. to pt.; $\frac{1}{10}$ oz. to gr.; $\frac{3}{100}$ lb. to oz.
18. $\frac{3}{11}$ of a T. to lb.; $\frac{1}{1000}$ cwt. to oz.; $\frac{9}{50}$ bu. to qt.
19. .007 of a ton to pounds.

SOLUTION.

.007 T.
20
.140 cwt.
100
14.000 lb.

EXPLANATION.—Since 1 ton is 20 hundred-weight, .007 of a ton are .007 times 20 cwt., or .140 of a hundred-weight.

Since 1 hundred-weight is 100 pounds, .140 of a cwt. are .140 times 100 pounds, or 14 pounds. Hence, .007 of a ton equals *14 lb.*

20. .009 yr. to hr.; .018 da. to sec.; .025 leap yr. to hr.

21. .5 of a ream to sheets; .0075° to ″; .15 grt. gr. to doz.

22. $.005 to mills; £.075 to far.; 4.5 mi. to rd. and to ft.

23. .0625 hhd. to qt.; .008 gal. to gills; .02½ bbl. to qt.

24. .001 A. to sq. ft.; .02¼ A. to sq. ch.; .05 sq. rd. to sq. in.

25. How many feet long is the telegraph-wire between Philadelphia and New York, the distance being 96 miles?

26. If a boy buys a bushel of pea-nuts for $2.50, and sells them at $.05 a pint, what does he clear? At 6¼ cents?

27. On one shirt are 6 buttons. How many shirts will 72 gross of buttons trim? 60 dozen?

28. A stationer bought 5 reams of paper at $3.50 a ream, and sold it at 1 cent a sheet. What did he gain?

29. How much younger is a man who is 50 years old, than one who is 3 score and 10 years old?

30. If your pulse beats 75 times in a minute, how often does it beat in a day? In a week? A common year?

31. How many jugs, each containing 2 qt., can be filled from a barrel of vinegar? From a hogshead?

32. From 4 oz. 3 dr. 2 scr. of calomel, how many pills, each weighing 5 grains, can be made? Each 4½ gr.?

33. How many papers, each weighing .25 of a pound, can be filled from 5 cwt. 5 lb. of garden-seed?

34. If 1 pk. of clover-seed is sown on an acre of land, how many acres will 4.5 bushels seed?

35. If one pound sterling is worth $4.866½ in United States money, how much are £10 worth?

36. How much are 10 gal. 2 qt. of maple-syrup worth at $.20 a quart? At $.31¼? At $.37½? At $.50 a gallon?

CASE II.

Reduction Ascending.

1. How many feet in 36 in.? How many yards?

2. Change 48 far. to shillings; 200 ct. to dollars.

3. In 31 quarts, how many pecks?

418. Analysis. — Since 8 quarts are 1 peck, 31 quarts are as many pecks as the number of times 8 quarts are contained in 31 quarts, or 3 pecks and 7 quarts remaining. Hence, etc.

4. How many gallons are 22 pints? 59 pints? 37 pints?

5. Change 51 gi. to pints. To quarts. To qt. pt. gi.

6. At $2.56 for 32 qt. of berries, what is the cost of 2 bu.?

7. What part of a pound Troy are 6 oz.? 8 oz.? 10 oz.?

8. At 24 cents a yard, what will 2 ft. of ribbon cost? What will 18 in. cost? 12 in.? 9 in.? 8 in.? 6 in.?

419. Principle.

A denominate number is changed to higher denominations by division.

Written Exercises.

420. Example.—Change 608 ft. to higher denominations.

SOLUTION.

3 ft.) 608 ft.

$5\frac{1}{2}$ yd.) 202 yd. + 2 ft.

2 | 2

11) 404 half-yd.

36 rd. + $\frac{8}{2}$ yd. = 4 yd.

608 ft. = 36 rd. 4 yd. 2 ft.

EXPLANATION. — Since 3 feet are 1 yard, 608 ft. are as many yards as the number of times 3 ft. are contained in 608 ft., or 202 times with 2 remaining, or 202 yd. 2 ft.

Since $5\frac{1}{2}$ yd. are 1 rod, 202 yd. are as many rods as the number of times $5\frac{1}{2}$ yd. are contained in 202 yd. $5\frac{1}{2}$ yd. changed to half-yards are 11 half-yards, and 202 yd. are 404 half-yards. 11 half-yards are contained in 404 half-yards 36 times with 8 half-yards, or 4 yards, remaining. Hence, 608 feet are *36 rd. 4 yd. 2 ft.*, the higher denominations required.

421. Rule for Reduction Ascending.

I. *Divide the given number by the number of units of its own denomination which equals one of the next higher, and write the remainder, if any.*

II. *In the same manner change this result to the next higher*

denomination; and so proceed until the given number is changed to the denomination required.

III. *The last quotient, and the remainders, if any, written in their order from the highest denomination to the lowest, will be the result required.*

Written Exercises.

Change to integers of higher denominations —

1. 756 in. to yards; 792 ft. to rods; 960 rd. to miles.
2. 3620 in. to rods; 1220 feet to mi.; 1000000 in.
3. 864 sq. in. to square feet; 1089 sq. ft. to square yards.
4. 65340 sq. ft. to square rods; 26620 sq. yd. to acres.
5. 100000 sq. in. to A.; 200000 sq. ft.; 2000000 sq. yd.
6. 15552 cu. in. to cubic feet; 3375 cu. ft. to cubic yards.
7. 45678 cu. in. to cu. yd.; 7986 cu. ft.; 46356 cu. in.
8. 123 gi. to quarts; 756 gi. to gallons; 1234 pt. to gallons.
9. 1000 gi. to gal.; 2000 pt. to bbl.; 30000 gi. to hhd.
10. 136 pt. to pecks; 320 qt. to pecks, and to bushels.
11. 2016 pwt. to lb.; 200 oz.; 10000 gr.; 200000 gr.
12. 3200 oz. to hundred-weight; 5000 lbs. to cwt.; to tons.
13. 20000 oz. to T.; 30000 lb.; 1000000 oz.; 4567800 lb.
14. 3600 sec. to hours; 10000 min. to days; 367 da. to yr.
15. 10000 sec.; 20000 min.; 30000 hr.; 1000000 min.
16. 2 feet to rods.

SOLUTION.

$2\,ft. = \frac{2}{1} \div 3 = \frac{2}{1} \times \frac{1}{3} = \frac{2}{3}\,yd.$

$\frac{2}{3}\,yd. = \frac{2}{3} \div \frac{11}{2} = \frac{2}{3} \times \frac{2}{11} = \frac{4}{33}\,rd.$

EXPLANATION.—Since 3 feet are 1 yard, there is $\frac{1}{3}$ as many yards as feet, or $\frac{2}{3}$ of a yard. Since $5\frac{1}{2}$, or $\frac{11}{2}$ of a yard are 1 rod, there are $\frac{2}{11}$ as many rods as yards, or $\frac{4}{33}$ of a yard.

17. 144 in. to ft.; to yd.; to rd.; to mi. 55 yd. to mi.
18. 144 sq. in. to sq. ft.; to sq. yd.; to sq. rd. 8 sq. ch. to A.
19. 2000 cu. in. to cu. ft.; to cu. yd.; to cd. 36 cu. ft. to cd.

20. $\frac{3}{4}$ of a gi. to pt.; to qt.; to gal. $\frac{7}{8}$ qt. to pk.; to bu.

21. $4\frac{3}{10}$ of a pk. to bu.; $10\frac{1}{2}$ gr. to oz.; $12\frac{1}{2}$ min. to hr.

22. 12 pwt. to the decimal of a pound Troy.

SOLUTION.
12 pwt. $= 12 \div 20 = .6$ *oz.*
$.6$ *oz.* $= .6 \div 12 = .05$ *lb.*

EXPLANATION. — Since 20 pennyweights are 1 ounce, there is $\frac{1}{20}$ as many ounces as pennyweights, or .6 oz. Since 12 ounces are 1 lb., there is $\frac{1}{12}$ as many pounds as ounces, or *.05 lb.*, the decimal required.

23. 20 gr. to oz.; to lb.; 24 ℈ to oz.; 6 oz. to lb.

24. 250 oz. to cwt.; 25 lb. to cwt.; to T.; 8 cwt. to T.

25. 5 mills to $; 3 qr. to s.; to £; 8 s. to £; 10 s. 6 d. to £.

26. $60\frac{1}{2}$ sec. to hr.; to days; $3\frac{1}{2}$ da. to wk.; to years.

27. 110 in. to rd.; 55 yd. to mi.; 121 sq. in. to sq. yd.; $10\frac{3}{4}$ sq. rd. to A.; 144 cu. in. to cu. ft.; $3\frac{1}{2}$ cu. ft. to cu. yd.

28. If sound travels 1142 feet in a second of time, how many miles will it travel in a minute?

29. How long will it take a clock, ticking once every second, to tick a million times? To tick 3050500 times?

30. If a rifle-ball weighs an ounce, how much lead is needed to cast 50000 rifle-balls? To cast 1000000 balls?

31. A carter feeds 6 quarts of shelled corn a day. How much must he buy for his horse in a year? A leap year?

32. At $1.50 a rod, how many miles of fence can be made for $900? For $1200? For $2250? For $5000?

33. How many ounces of calomel are needed to make 3 gross of pills, each weighing 5 grains?

34. If 47 gal. 1 qt. of vinegar be put into barrels, and sold at $10\frac{1}{2}$ a barrel, what will be received for it?

35. The circumference of the earth is about 25000 miles. At 4 miles an hour, how long will it take to walk that distance?

36. At 10 cents a quart, how many bushels of cherries can be bought for $7.50? For $11.25?

SECTION XXII.

ADDITION OF COMPOUND NUMBERS.

422. Addition, subtraction, multiplication, and division of compound numbers depend upon the same general principles that govern similar operations in simple numbers.

Written Exercises.

423. *Example.* — Find the sum of 3 rd. 4 yd. 10 in., 4 yd. 2 ft. 8 in., 7 rd. 5 yd. 1 ft., and 3 rd. 3 yd. 2 ft. 8 in.

SOLUTION.

3 rd.	*4 yd.*	*0 ft.*	*10 in.*
0	*4*	*2*	*8*
7	*5*	*1*	*0*
3	*3*	*2*	*8*
16	$1\frac{1}{2}$	*1*	*2*
	$\frac{1}{2} = 1$		*6*

16 rd. 1 yd. 2 ft. 8 in.

EXPLANATION. — Since only like orders of units can be added, write the numbers so that units of the same denomination shall stand in the same column (81).

Begin with the lowest denomination, and add each column separately.

The sum of the inches is 26 inches, or 2 feet 2 in. Write 2 inches under the column of inches, and add 2 feet with the column of feet.

The sum of the feet is 7 feet, or 2 yd. 1 foot. Write 1 foot, etc.

The sum of the yards is 18 yd., or 3 rd. $1\frac{1}{2}$ yd. Write $1\frac{1}{2}$ yd., etc.

The sum of the rods is 16 rods, which write under the column of rods.

1 half-yard changed to lower denominations equals 1 foot 6 in., which, added to 1 foot 2 inches, gives the entire sum, *16 rd. 1 yd. 2 ft. 8 in.*

424. Rule for Addition of Compound Numbers.

I. *Write the compound numbers so that units of the same denomination shall stand in the same column.*

II. *Begin with the lowest denomination, and add each column separately.*

III. *If the sum is less than one of the next higher denomination, write it under the column added.*

If the sum is equal to, or greater than, one of the next higher denomination, change it to that higher denomination,

write the remainder under the column added, and add the ones of the higher denomination with the column of that denomination.

Problems.

(1)			(2)		(3)		
10 mi.	*50 rd.*	*4 yd.*	*25 A.*	*90 sq. rd.*	*15 gals.*	*3 qt.*	*1 pt.*
8	*25*	*3*	*50*	*100*	*27*	*2*	*0*
7	*20*	*2*	*75*	*50*	*59*	*3*	*1*
25	*10*	*5*	*100*	*87*	*170*	*2*	*1*

Find the sum of—

4. 10 mi. 75 rd. 5 yd. 2 ft.; 150 rd. 3 ft.; 100 mi. 100 rd.

5. 75 mi. 200 rd. 5 yd.; 200 mi. 4 yd.; 95 mi. 2 ft.

6. 75 A. 10 sq. rd.; 100 sq. rd. 5 sq. ft.; 130 A. 10 sq. yd.

7. 175 A. 30 sq. yd.; 98 A. 8 sq. ft.; 765 A. 25 sq. yd.

8. 99 cu. yd. 20 cu. ft.; 25 cu. ft. 88 cu. in.; 500 cu. yd. 1700 cu. in.; 44 cu. yd. 30 cu. ft. 732 cu. in.

9. 85 T. 75 lb.; 137 T. 15 cwt. $10\frac{1}{2}$ oz.; 76 T. 50 lb.; 876 T. 19 cwt. $8\frac{1}{4}$ oz.; and 976 T. 18 cwt. 85 lb. $8\frac{1}{8}$ oz.

10. 10 T. 4 cwt. 90 lb.; 50 T. 95 lb. 8 oz.; 100 T. 10 cwt.; 500 T. 2 cwt. 90 lb.; 20 cwt. 80 lb. 12 oz.; 90 lb. $15\frac{3}{4}$ oz.

11. Add 11 lb. 3 oz. 2 pwt.; 10 oz. 18 gr.; 15 lb. 5 pwt. 10 gr.; 25 lb. 6 oz. 15 pwt.; 8 lbs. 21 oz. 15 pwt. $18\frac{2}{3}$ gr.

12. 100 cd. 8 cu. ft.; 500 cd. 17 cu. ft. 1400 cu. in.; 210 cd. 20 cu. ft.; 1000 cd. 25 cu. ft. 1700.25 cu. in.

13. How much coal are in five cars containing 17 T. 10 cwt. 73 lb. of coal; 15 T. 15 cwt. 80 lb.; 18 T. 99 lb.; 17 T. 18 cwt.; and 16 T. 10 cwt. 75 lb.?

14. Philadelphia is in longitude 75° 8′ 42″ W., and Rome 12° 27′ E. How far apart are they?

15. Washington is in latitude 38° 53′ 20″ N., and Rio Janeiro 22° 56′ S. How many degrees between them?

16. In a farm there are three fields: the first contains 10 A. 40 sq. rd.; the second 25 A. 30 sq. yd.; and the third 20 A. 100 sq. rd. 20 sq. yd. How large is the farm?

SECTION XXIII.

SUBTRACTION OF COMPOUND NUMBERS.

Written Exercises.

425. *Example.*— From 25 mi. 80 rd. 4 yd. take 16 mi. 100 rd. 5 yd.

SOLUTION.

25 mi.	*80 rd.*	*4 yd.*
16	*100*	*5*
8	*299*	$4\frac{1}{2}$
		$\frac{1}{2}$ = *1 ft. 6 in.*
8 mi.	*299 rd.*	*4 yd. 1 ft. 6 in.*

EXPLANATION.—Since only like orders of units can be taken one from another, write the numbers so that units of the same denomination shall stand in the same column.

Begin with the lowest denomination, and subtract each column separately.

Since 5 yd. cannot be taken from 4 yd., take 1 rod, or $5\frac{1}{2}$ yd., from 80 rd., leaving 79 rd., and add the $5\frac{1}{2}$ yd. borrowed to the 4 yd., making $9\frac{1}{2}$ yd. 5 yd. from $9\frac{1}{2}$ yd. are $4\frac{1}{2}$ yd., which write under the column of yards.

Since 100 rods cannot be taken from 79 rods, take 1 mile, or 320 rods, from 25 miles, etc.

16 miles from 24 miles are 8 miles, which write, etc.

1 half-yard changed to lower denomination equals 1 foot 6 in., which gives the entire difference, *8 mi. 299 rd. 4 yd. 1 ft. 6 in.*

426. Rule for Subtraction of Compound Numbers.

I. *Write the subtrahend under the minuend so that units of the same denomination shall stand in the same column.*

II. *Begin with the lowest denomination, and subtract each column separately.*

III. *If the units of any denomination of the subtrahend are less than the units of the same denomination in the minuend, write the difference under the denomination subtracted.*

If the units of any denomination of the subtrahend are greater than the units of the same denomination in the minu-

end, add to the minuend as many units as equal one of the next higher denomination, and subtract; then consider the units of that higher denomination of the minuend as one less, and subtract as before.

Problems.

(1)

7 mi.	75 rd.	3 yd.
3	100	5

(2)

90 A.	7 sq. rd.	10 sq. yd.
75	100	30

(3)

40 bu.	1 pk.	6 qt.
37	3	7

(4)

90 rd.	3 yd.	1 ft.
75	5	2

(5)

75 T.	15 cwt.	90 lb.
69	18	95

(6)

30 sq. rd.	20 sq. yd.	6 sq. ft.
25	25	8

Find the remainder of—

7. 7 da. 5 hr. 50 min. 55 sec. — 5 da. 59 min. 57 sec.

8. 150 T. 15 cwt. 25 lb. — 100 T. 18 cwt. 50 lb.

9. 75 hhd. 50 gal. 2 qt. 3 gi. — 56 hhd. 60 gal. 1 pt.

10. 60° 10′ 30″ — 55° 15′ 45″; £100 — £50 9 s. 10 d.

11. 100 mi. 180 rd.— 90 mi. 250 rd. 5 yd. 2 ft.

12. Boston is in latitude 42° 21′ 30″ N., and New York in 40° 42′ 43″ N. What is the difference in their latitude?

13. Washington is in longitude 77° 0′ 15″ W., and San Francisco in 122° 46′ 48″ W. What is the difference in their longitude? Take the difference from 180°.

14. The sum of two compound numbers is 400 mi., and one of them is 176 mi. 200 rd. 2 ft. What is the other?

15. The minuend is 47 A. 30 sq. rd. 6 sq. ft., and the remainder is 29 A. 30 sq. yd. 8 sq. ft. Find the subtrahend.

16. Of 16 mi. 5 rd. 2 yd. 1 ft. of a telegraph line, 13 mi. 5 yd. 2 ft. are finished. What is yet to be put up?

17. From a farm containing 100 A. 30 sq. rd., there were sold 50 A. 50 sq. rd. 30 sq. yd. What remained?

18. How many years, months, and days from December 14, 1799, to July 4, 1876?

SOLUTION.

1876 yr. 7 mo. 4 da.
1799 12 14

76 yr. 6 mo. 20 da.

EXPLANATION. — Since the later of two dates expresses a greater period of time than the earlier, write it as the minuend, and the earlier as the subtrahend, placing to the right of each year the number of the month, and the number of the day. Consider 30 days as a month, and 12 months as a year, and subtract (425).

19. How long from Apr. 13, 1899, to Jan. 1, 1901? How long from May 10, 1900, to Dec. 25, 1901?

20. A note given July 10, 1900, was paid Mar. 1, 1901. How long had it run? If paid Sept. 9, 1901?

21. The sum of three numbers is 200 A. 10 sq. rd., and two of them are 98 A. 60 sq. rd. 8 sq. ft., and 89 A. 30 sq. yd. 72 sq. in. What is the other?

22. A contractor engaged to build 75 mi. of telegraph; after building 25 mi. 250 rd., a storm blew down 2 mi. 10 rd. 2 ft. What remained to be done?

23. From a piece of land containing 100 A. 150 sq. rd., there were sold at one time 25 A. 30 sq. rd., and at another time 50 A. 10 sq. rd. 5 sq. ft. How much remained?

SECTION XXIV.

MULTIPLICATION OF COMPOUND NUMBERS.

Written Exercises.

427. *Example.* — Find the product of 4 miles 100 rods 4 yards multiplied by 6.

SOLUTION.

4 mi. 100 rd. 4 yd.
6

25 mi. 284 rd. 2 yd.

EXPLANATION.—Write the multiplier, etc. Begin with the lowest denomination, and multiply the units of each denomination separately.

Six times 4 yd. are 24 yd., or 4 rd. 2 yd.; write 2 yd. under yards, and carry 4 rd. to add to the product of the rods.

Six times 100 rd. are 600 rd.; 600 rd. and 4 rd. are 604 rd., or 1 mi. 284 rd. Write 284 rd., etc.

Six times 4 mi. are 24 mi.; 24 mi. and 1 mi. are 25 mi., which write under miles, giving the entire product *25 mi. 284 rd. 2 yd.*

428. Rule for Multiplication of Compound Numbers.

I. *Write the multiplier under the lowest denomination of the multiplicand.*

II. *Begin with the lowest denomination, and multiply the units of each denomination separately.*

III. *If the product is less than one of the next higher denomination, write it under the denomination multiplied.*

If the product is equal to, or greater than, one of the next higher denomination, change it to that higher denomination, write the remainder under the denomination multiplied, and add the ones of the higher denomination to the product of that denomination.

Problems.

(1)

10 mi. 40 rd. 4 yd.
5

(2)

75 A. 10 sq. yd.
9

(3)

7 gal. 2 qt. 1 pt.
13

(4)

275 A. 80 sq. rd. 8 sq. ft. 70 sq. in.
75

(5)

345 mi. 40 rd. 5 yd. 2 ft. 10 in.
98

Find the product of—

6. 7 da. 5 hr. 50 min. × 5; by 7; by 9; by 11; by 25.
7. 10° 15′ 35″ × 6; by 8; by 10; by 12; by 20; by 75.
8. £10 10 s. 8½ d. × 3; by 6; by 9; by 12; by 110.
9. 5 T. 3 cwt. 90 lb. × 4; by 9; by 14; by 35; by 100.
10. 10 lb. 4 oz. 15 pwt. 18 gr. × 7; by 12; by 17; by 125.
11. 85 A. 25 sq. rd. 8 sq. ft. × 25; by 46; by 127; by 176.

12. 98 mi. 80 rd. 2 ft. × 154; by 234; by 345; by 567.

13. If an engine is run at the average rate of 25 mi. 10 rd. 5 yd. per hour, how far does it run in 12 hours?

14. If a team can haul 1 cord $20\frac{1}{2}$ cu. ft. of wood in one load, how much can it haul in 50 loads? In 75?

15. How much time is there in 100 solar years, each 365 da. 5 hr. 48 min. 49.7 sec.? In 150? In 175? In 234?

16. If the average weight of a bushel of wheat is 59 lb. $8\frac{1}{2}$ oz., what is the weight of 50 bushels? Of 65? 78?

17. A piece of land was divided into 25 lots, each 75 sq. rd. 30 sq. yd. How much land was in the piece?

18. From 16 mi. 8 rd. 4 yd. 1 ft. take 10 mi. 20 rd. 5 yd., and multiply the remainder by 9; by 10; by 15.

19. A farmer sold 40 A. 100 sq. rd. from a farm containing 60 A. 15 sq. rd. 3 sq. yd., and bought 4 times as much as the remainder. How much land had he then?

20. Of 125 mi. 80 rd. 5 yd. of railroad, 4 sections, each 25 mi. 100 rd. 4 yd., have been finished. How much remains to be done?

SECTION XXV.

DIVISION OF COMPOUND NUMBERS.

Written Exercises.

429. *Example* 1. — Divide 41 mi. 20 rd. 5 yd. by 8.

SOLUTION.

8)41 mi. 20 rd. 5 yd.

5 mi. 42 rd. 3 $\frac{5}{8}$ yd.

EXPLANATION. — Write the divisor, etc. Begin with the highest denomination, and divide the units of each denomination separately.

One eighth of 41 mi. is 5 mi. with a remainder of 1 mi. Write 5 mi. as the miles of the quotient, and change the 1 mi. to rods. 1 mi. is 320 rd., and 320 rods plus 20 rd. are 340 rd.

One eighth of 340 rd. is 42 rd. with a remainder of 4 rd. Write 42 rd. as the rods of the quotient, and change the 4 rd. to yards, etc.

One eighth of 27 yd. is 3 yd. with a remainder of 3 yd., or $3\frac{3}{8}$ yd., which write, etc.; giving the entire quotient *5 mi. 42 rd. $3\frac{3}{8}$ yd.*

430. *Example* 2. — 138 gal. 1 pt. ÷ 27 gal. 2 qt. 1 pt.

SOLUTION.

138 gal. 1 pt. = 1105 pt.
27 gal. 2 qt. 1 pt. = 221 pt.
1105 pt. ÷ 221 pt. = 5

EXPLANATION. — Since the dividend and the divisor are similar numbers, the quotient may be most easily found by changing both compound numbers to the lowest denomination mentioned in either, and then dividing as in simple numbers.

138 gal. 1 pt. equal 1105 pt., and 27 gal. 2 qt. 1 pt. equal 221 pt.

221 pt. are contained in 1105 pt. *5* times.

431. Rules for Division of Compound Numbers.

When the divisor is a simple number,

I. *Write the divisor at the left of the dividend.*

II. *Begin with the highest denomination, and divide the units of each denomination separately.*

III. *If after dividing the number of any denomination there is a remainder, change it to the next lower denomination, and to the result add the given number of that denomination; then divide as before. The several results will be the quotient required.*

When the divisor is a compound number,

Change the dividend and the divisor to the lowest denomination mentioned in either, and then divide as in simple numbers.

Problems.

(1) *7)10 mi. 8 rd. 3 yd.*

(2) *8)20 A. 25 sq. rd.*

(3) *9)50 bu. 2 pk. 6 qt.*

(4) *10)25 cd. 10 cu. ft. 100 cu. in.*

(5) *12)7 mi. 100 rd. 5 yd. 2 ft. 10 in.*

Find the quotient of —

6. 12 gr. gro. 7 gro. 10 doz. ÷ 6; 15 rm. 10 sheets ÷ 7.

7. 15 yr. 5 hr. ÷ 10; 365 da. 5 hr. 48 min. 49 sec. ÷ 11.

8. 90° 30″ ÷ 12; £15 10 s. $8\frac{1}{2}$ d. ÷ 13; £100 6 d. ÷ 14.

9. 75 lb. 10 pwt. $12\frac{1}{2}$ gr. $\div$ 15; 10 oz. 7 pwt. 20 gr. $\div$ 16.

10. 100 T. 15 cwt. 75 lb. $\div$ 18; 18 cwt. 90 lb. 10 oz. $\div$ 20.

11. 170 A. 19 sq. rd. 11 sq. yd. $\div$ 46; by 65; by 78; by 85; by 97; by 105; by 126; by 154; by 175; by 250.

12. If 9 loads of hay weigh 10 T. 15 cwt. 75 lb., what is the average weight? If 10 loads? 11 loads?

13. Divide 98 mi. 60 rd. 5 yd. 2 ft. 6 in. by 45; by 54; by 67; by 85; by 98; by 106; by 125; by 150; by 199.

14. If 9 hhd. of sugar weigh 5 T. 6 cwt. $25\frac{3}{4}$ lb., what is the average weight? If they weigh 6 T. 10 cwt. 50 lb.?

15. How much is $\frac{2}{3}$ of 15 lb. 10 oz. 2 scr. 15 gr.? $\frac{3}{5}$?

16. Find $\frac{3}{4}$ of 10 mi. 4 rd. 1 ft.; $\frac{4}{5}$; $\frac{5}{6}$; $\frac{7}{8}$; $\frac{8}{9}$; $\frac{9}{10}$; $\frac{10}{11}$; $\frac{11}{12}$.

17. Add 10 A. 125 sq. rd. 20 sq. yd. and 25 A. 200 sq. rd. 15 sq. yd. 8 sq. ft., and divide the sum by 9; by 12; by 25.

18. From the sum of 40 gal. 2 qt. 2 gi., and 20 gal. 1 pt. take 15 gal. 1 qt. 3 gi., and divide the result by 15; by 18.

19. To the difference between 21 sq. rd. 18 sq. yd. 5 sq. ft. and 15 sq. rd. 8 sq. ft. add their sum, and divide by 15.

20. How many bottles, each holding 3 qt. 1 pt., can be filled from a cask containing 31 gal. 2 qt.?

21. How many baskets of peaches, each 2 pk. 4 qt., will make $3\frac{1}{2}$ bu.? $5\frac{1}{4}$ bu.? 7 bu.? $7\frac{1}{2}$ bu.? $10\frac{1}{4}$ bu.?

22. If a man steps 2 ft. 4 in., how many steps will he take in walking $\frac{3}{4}$ of a mile? $2\frac{1}{2}$ mi.? $\frac{7}{8}$ mi.? $3\frac{1}{4}$ mi.?

23. If a train of cars runs 25 mi. 60 rd. in an hour, in what time can it run 151 mi. 40 rd.?

24. If 8 hogsheads of molasses, each 42 gal. 2 qt., sell for $\$136.71\frac{7}{8}$, what is the selling price per gallon?

25. The sum of three numbers is 75 mi., and two of them are 25 mi. 275 rd. and 30 mi. 5 yd. Find the third.

26. Subtract the quotient of 21 A. 35 sq. yd. 30 sq. in. $\div$ 75, from the product of 21 A. 35 sq. yd. 30 sq. in. $\times$ 75, and divide the remainder by 225.

SECTION XXVI.

MEASUREMENTS.

CASE I.

Measurement of Surfaces.

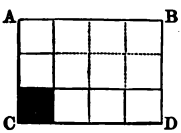
432. A ***Rectangle*** is a flat surface which has four straight sides and four square corners.

A rectangle has two dimensions: *length* and *breadth*.

433. The ***Area*** of a rectangle is the surface bounded by the sides of the rectangle.

Thus, the area of a blackboard is the surface bounded by the sides or edges of the blackboard.

Let the figure A B C D represent a surface 4 in. long and 3 in. wide. Now, there can be placed along the length of the surface as many square inches as there are inches in the length, making a row of 4 sq. in.; and there can be as many rows of square inches as there are inches in the width, making 3 rows. Hence, the surface contains 3 rows of 4 sq. in. each; and 3 times 4 sq. in., or *12 sq. in.*, must be the area of the surface.



434. Principle.

The area of a rectangle is the product of the length and the breadth.

Written Exercises.

435. *Example* 1. — How many square feet are in a floor $16\frac{2}{3}$ ft. long and $10\frac{1}{2}$ ft. wide?

SOLUTION.

Length $\times$ *breadth* = *area*.

$16\frac{2}{3}$ *ft.* $\times$ $10\frac{1}{2}$ *ft.* $= \overset{25}{\cancel{50}} \times \frac{21}{\cancel{2}} = 175$ *sq. ft.*

EXPLANATION.—Since the area of a rectangle is the product of the length and the breadth, the product of the length, $16\frac{2}{3}$ ft., multiplied by the breadth, $10\frac{1}{2}$ ft., must be *175 sq. ft.*, the area required.

436. *Example* 2. — A lot of ground 18 ft. wide contains 181 sq. yd. How long is it?

SOLUTION.

9 sq. ft. $\times$ *181* = *1629 sq. ft.*

Area $\div$ *breadth* = *length.*

1629 sq. ft. $\div$ *18 ft.* = $90\frac{1}{2}$ *ft.*

EXPLANATION. — 181 sq. yd. equal 1629 sq. ft.

Since the area is the product of the length and the breadth, the quotient of the area, 1629 sq. ft., divided by the breadth, 18 ft., must be $90\frac{1}{2}$ *ft.*, the length required.

437. Rules for the Measurement of Surfaces.

I. *Multiply the length by the breadth. The product will be the area of the surface required.*

II. *Divide the area of the surface by the length. The quotient will be the breadth required.*

III. *Divide the area of the surface by the breadth. The quotient will be the length required.*

Note 1. — When the length and the breadth are given to find the area, two factors are given to find their product.

2. When the area and one dimension are given to find the other dimension, the product of two factors and one factor are given to find the other factor.

Problems.

Find the areas of the surfaces having the following dimensions, and change each result to higher denominations:—

1. 7 in. by 6 in.; 8 in. by 7 in.
2. 8 ft. by 5 ft.; 9 ft. by 8 ft.
3. 10 ft. by $6\frac{1}{2}$ ft.; $15\frac{1}{3}$ ft. by 8 ft.
4. 18 ft. 9 in. by 15 ft.
5. 30 yd. by 20 yd. 2 ft.
6. 45.5 rd. by $10\frac{1}{5}$ rd.
7. How many sq. in. in a window-pane $18\frac{1}{2}$ in. by $12\frac{1}{4}$ in.?
8. How many sq. ft. of surface in a board 10 ft. long, 8 in. wide? In a board 12 ft. 9 in. long, $10\frac{1}{2}$ in. wide?
9. Find the area of a cellar of a square building, each side of which is 30 ft. 6 in. What is the area in sq. yd.?
10. In a village lot 90 ft. by 200 ft. are how many sq. yd.? How much is it worth at $1.25 a sq. rod?

Find the required dimension of the following surfaces: —

11. Area 437½ sq. in., breadth 17½ in.; breadth 18¾ in.

12. Area 762.5 sq. ft., length 30.5 ft.; length 24.4 ft.

13. Area 2318 sq. yd., length 30⅔ yd.; length 25 yd. 9 in.

14. Area 2 acres, breadth 30.25 rd.; breadth 20 yd. 2 ft.

15. Of a blackboard 7 ft. long, the area being 24½ sq. ft.?

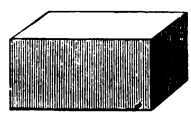
16. How long is a window-curtain 3½ ft. wide, if its surface is 2 sq. yd. 3 sq. ft.? If its surface is 3½ sq. yd.?

17. I have a lot which is 6 rods wide, and contains 77 sq. rd. How many rods of fence are needed to inclose it? How many yd.? Feet?

CASE II.

Measurement of Solids.

438. A ***Rectangular Solid*** is a body which is bounded by six flat surfaces having square corners.

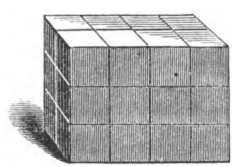


A rectangular solid has three dimensions: *length, breadth,* and *thickness.*

439. The ***Solidity,*** or ***Capacity,*** of a solid is the space bounded by the surfaces of the solid.

Thus, the solidity of a block, or the capacity of a box, is the space bounded by the surfaces of the block or the box.

Let the figure represent a solid 4 in. long, 2 in. wide, and 3 in. high.



Now, there can be placed along the length of the solid as many cubic inches as there are inches in the length, making a row of 4 cu. in.; and there can be as many rows of cubic inches on the lower surface as there are inches in the width, making 2 rows of 4 cu. in. each, which form one layer of 2 times 4 cubic inches, or 8 cu. in. Also, there can be as many layers as there are inches in the height, making 3 layers.

Hence, the solid contains 3 layers of 8 cu. in. each; and 3 times 2 times 4 cu. in., or ***24 cu. in.,*** must be the solidity of the solid.

440. Principle.

The solidity of a rectangular solid is the continued product of the length, the breadth, and the thickness.

Written Exercises.

441. *Example* 1.—Find the solidity of a block of granite $7\frac{1}{3}$ ft. long, $2\frac{1}{4}$ ft. wide, and 2 ft. thick.

SOLUTION.

Length × *breadth* × *thickness* = *solidity*.

$7\frac{1}{3}$ *ft.* × $2\frac{1}{4}$ *ft.* × 2 *ft.* = 33 *cu. ft.*

$\frac{22}{3} \times \frac{9}{4} \times \frac{2}{1} = 33.$

EXPLANATION. — Since the solidity of a solid is the continued product of the length, the breadth, and the thickness, the continued product of the length, $7\frac{1}{3}$ ft., by the breadth, $2\frac{1}{4}$ ft., by the thickness, 2 ft., must give 33 *cu. ft.*, the solidity required.

442. *Example* 2. — A pile of stone $8\frac{1}{3}$ ft. long and $6\frac{3}{4}$ ft. wide contains 225 cu. ft. How high is it?

SOLUTION.

Solidity ÷ (*length* × *breadth*) = *height*.

225 *cu. ft.* ÷ ($8\frac{1}{3}$ *ft.* × $6\frac{3}{4}$ *ft.*) = 4 *ft.*

$\frac{225}{1} \times \frac{3}{25} \times \frac{4}{27} = 4.$

EXPLANATION.—Since the solidity is the continued product of the length, the breadth, and the thickness, or height, the quotient of the solidity, 225 cu. ft., divided by the product of the length, $8\frac{1}{3}$ ft., by the width, $6\frac{3}{4}$ ft., must give 4 *ft.*, the height required.

443. Rules for the Measurement of Solids.

I. *Multiply the length, the breadth, and the thickness together. The product will be the solidity or capacity required.*

II. *Divide the solidity by the product of the length and the breadth. The quotient will be the thickness required.*

III. *Divide the solidity by the product of the length and the thickness. The quotient will be the breadth required.*

IV. *Divide the solidity by the product of the breadth and the thickness. The quotient will be the length required.*

Note 1. — When the length, the breadth, and the thickness are given to find the solidity, three factors are given to find their product.

2. When the solidity and two dimensions are given to find the other dimension, the product of three factors and two of the factors are given to find the other factor.

Problems.

Find the capacities of solids having the following dimensions, and change each result to higher denominations:—

1. 8 in. by 7 in. by $4\frac{1}{2}$ in.
2. $9\frac{3}{8}$ ft. by 8 ft. by 7 ft.
3. $12\frac{3}{4}$ ft. by $9\frac{1}{3}$ ft. by 8 ft.
4. 10 ft. 6 in. by $8\frac{1}{2}$ ft. by 7 ft.
5. 12.5 yd. by 9 yd. 2 ft. by 8 yd.
6. 20.4 rd. by $10\frac{1}{2}$ rd. by 8.5 rd.

7. Of a cellar 30.5 ft. long, 14 ft. wide, and $6\frac{1}{2}$ ft. deep.

8. Of a packing-box 6 ft. 8 in. long, $4\frac{1}{2}$ ft. deep, 3 ft. wide.

9. Of a bin 12 ft. 9 in. long, 7.5 ft. wide, and $4\frac{1}{3}$ ft. deep.

10. Of a thousand bricks, each 8 in., by 4 in., by 2 in.

11. How many cords are in a pile of wood 90 ft. 6 in. long, 8 ft. wide, and $5\frac{1}{2}$ ft. high?

Find the required dimension of the following solids:—

12. Solidity 459 cu. ft., length $8\frac{1}{2}$ ft., height 8 ft.

13. Solidity 1380 cu. yd., length 20.5 ft., depth 4 ft. 6 in.

14. If a packing-box 10 ft. 6 in. long and $5\frac{1}{2}$ ft. deep contains 462 cu. ft., how wide is it? If it contains 691 cu. ft.?

15. A mason used 768 cu. ft. of stone in building a wall 4 ft. high and 2 ft. thick. How long was it?

16. How many bricks 8 in. × 4 in. × 2 in. can be put in a cart 8 ft. by 4 ft. by 2 ft.?

Review.

1. Define ***Reduction of Denominate Numbers.*** How many kinds of reduction are there? What is ***Reduction Descending?*** State the principle of reduction descending. Repeat the rule. How are hundreds changed to tens? Tens to ones? What is the uniform multiplier in changing simple integers to lower orders? Are the multipliers in compound numbers uniform, or varying?

2. Define ***Reduction Ascending.*** State the principles of reduction ascending. Repeat the rule. How are ones changed to tens? Tens to hundreds? Hundreds to thousands? What is the uniform divisor in changing simple integers to higher orders of units? Are the divisors in compound numbers uniform, or varying?

3. What is meant by ***Addition of Compound Numbers?*** Subtraction of compound numbers? Multiplication of compound numbers? Division of compound numbers? Upon what general principles do operations upon compound numbers depend? Repeat each rule.

4. Define ***Rectangle.*** What dimensions has a rectangle? Define area, and explain how it may be found. State the principle. Repeat the rules to find the area and each dimension of a rectangle. Define ***Rectangular Solid.*** What dimensions has a rectangular solid? Define solidity, or capacity, and explain how it may be found. State the principle. Repeat the rules to find the solidity and each dimension of a rectangular solid.

General Review Problems.

1. Multiply $(800 + \overline{45 \times 75}) \div 167$ by $750 - \overline{11875 \div 95}$.

2. Divide $900 + \overline{25 \times 25 \times 64}$ by $\overline{625 \div 25} + \overline{84 \times 75}$.

3. From $\overline{480 \times 75} \div 90$ take $(125 \div 25) \times \overline{734 - 659}$.

4. Find the value of $(9\frac{3}{15} + 8\frac{3}{10}) \times 36 \div (75\frac{11}{18} - 54\frac{9}{10})$.

5. How much is $(95\frac{11}{12} - 86\frac{14}{15}) + \frac{99}{60} \times 7\frac{3}{11}$ times $\frac{5}{9}$?

6. Find the result of $(18.75 \times 48 \div .018) \div (125 \times .004)$.

7. The minuend is $\$187.37\frac{1}{2}$, and the subtrahend $\$137.66\frac{2}{3}$. What is the remainder? If the subtrahend is $\$75.062\frac{1}{2}$?

8. The sum of three numbers is $875.0075\frac{1}{2}$. If two of them are 750 and .000075, what is the third?

9. What is the product of the three factors $\frac{14}{17}$ of $13\frac{16}{21}$, 8 times $10\frac{11}{58}$, and $99\frac{10}{16}$ times $\frac{8}{85}$?

10. The product of three factors is 10 times $8\frac{2}{5}$, and two of them are 26 times $7\frac{12}{39}$ and $\overline{5\frac{5}{19} \times 8\frac{11}{50}}$. Find the third.

11. Find the quotient of $\frac{7}{13}$ of $19\frac{17}{21}$ divided by the sum of 7 times $8\frac{3}{11} + \frac{1}{16}$ of $44\frac{4}{5}$. Of $.0001 \div \overline{100.0695 - 100.007}$.

12. The remainder is $\$1.37\frac{1}{2}$, the divisor 975, and the quotient \$100.50. Find the dividend. If the divisor is .0975.

13. The divisor is 78, the quotient 325, and the remainder $88\frac{8}{52}$. What is $15\frac{7}{55}$ times the dividend?

14. The sum of four numbers is $1328\frac{2}{3}\frac{2}{9}$, and three of them are $237\frac{1}{2}\frac{9}{8}$, $456\frac{2}{7}\frac{3}{8}$, and $10\frac{1}{6}\frac{1}{5}$. What is the fourth?

15. The product of four factors is 1219, and three of them are 8 times $13\frac{7}{16}$, $\frac{40}{43}$ of $3\frac{3}{20}$, and $\frac{1}{14}\frac{3}{4} + 2\frac{1}{2}\frac{9}{1}$. Find the fourth.

16. What is $\overline{7 \div .007} + (700 \times .075)$? $\overline{800 + (.0008 \div 8 \times 80)}$? $400 + 10.0002 - 300.00003$.

17. From $13\frac{1}{20} \div 19\frac{17}{22}$ take $20\frac{5}{11}$ times $\frac{1}{75}$. Find the sum.

18. To 5000 times .0004 add 25000 ÷ 5 times .00005.

19. Multiply $21\frac{7}{20}$ by $7\frac{1}{5}$ times $\frac{10}{61}$, take $19\frac{11}{12}$ from the product, and to the remainder add $19\frac{19}{24}$.

20. Multiply the sum of $65\frac{10}{21}$ and $89\frac{9}{14}$ by the quotient of $121\frac{2}{13} \div 12\frac{3}{26}$. Divide the product by $1\frac{1}{4}\frac{3}{2}$.

21. Divide the product of 25 thousand times 25 thousandths by the quotient of 25 thousand ÷ 25 thousandths.

22. $\dfrac{8\frac{8}{9} \text{ times } 6\frac{18}{24} + 5\frac{5}{6}}{(22\frac{10}{11} \times \frac{7}{18}) + 15\frac{7}{22}} = \text{what?}$ $7\frac{9}{10} + \dfrac{\frac{1}{10} + \frac{1}{12}}{\frac{1}{15} - \frac{1}{20}}$?

23. $\dfrac{18\frac{3}{4} \times 64 \div 1\frac{9}{16}}{\frac{8}{7} \text{ of } 7\frac{7}{9} \text{ times } 192} = \text{what?}$ $\dfrac{\frac{1}{7} \times \frac{1}{8}}{\frac{1}{9} \div \frac{1}{10}} \div \dfrac{\frac{1}{5} + \frac{1}{6}}{\frac{1}{7} - \frac{1}{8}}$?

24. Change $\frac{7}{12}$ of $\frac{26}{35}$ of $\frac{55}{39}$ to 72ds, and change $\frac{30600}{165000}$, $\frac{17}{56}\frac{55}{160}\frac{50}{}$, and $\frac{14}{25}\frac{44}{93}\frac{55}{1}$ to their lowest terms.

25. Find the least common multiple of the first five prime odd numbers. The first five composite odd numbers.

26. Find the greatest number that exactly divides 21296 and 33528, and the smallest that exactly contains each.

27. Four partners put in business \$3750.75, $\$5000.37\frac{1}{2}$, \$4000.50, and $\$6250.18\frac{3}{4}$. At the end of the year the firm was worth \$22000. What were the average profits?

28. How many pieces of stone flagging, each $7\frac{1}{2}$ feet square, are required to make a sidewalk 120 ft. long 3 ft. 9 in. wide?

29. Divide $\frac{\frac{3}{14} \text{ of } 5\frac{3}{5}}{\frac{5}{3} - \frac{3}{5}} \times 12$ times $\frac{2\frac{3}{7}}{6\frac{3}{8}}$ by their sum.

30. Find the prime factors of 51968, 73008, and 86625.

31. Find the cost of $5\frac{3}{4}$ bales of cotton, each weighing 4.75 cwt., at $\$.13\frac{1}{2}$ per pound.

32. .00625 is $\frac{1}{12}$ of what number? $\frac{25}{28}$ of what?

33. How many cu. in. are in 37.5 bu.? How many bushels in 212730.025 cu. in.?

34. How many cubic feet of lumber are in a load of 25 planks, each 15 ft. long, 10 in. wide, $2\frac{1}{2}$ in. thick?

35. From a 63-gallon cask, $\frac{3}{4}$ full of wine, there were drawn $20\frac{2}{3}$ gall. What was the balance worth at \$2.75 per gallon?

36. At \$.5625 per yard, how much delaine can be bought for \$54, and how many dresses each $13\frac{5}{7}$ yd. can be made from it?

37. A farmer raised 343.75 bu. of corn from 11 acres, and 437.5 bu. from 14 acres. What was the average yield per acre?

38. At \$75 an acre, what is the cost of a piece of land 150 rd. wide and 225 rd. long? And at \$.75 a yard, how much will it cost to make a fence around it?

39. At \$10 a hundred, how much will 225 cherry-trees cost?

40. A person who had $2\frac{3}{5}$ times \$500, spent $\frac{2}{3}$ of it, and earned $2\frac{3}{4}$ times as much as remained. What had he then?

41. Divide 6125 by 5456875, and prove the result.

42. How many square yards in the walls of a room 30 ft. long, 18 ft. 4 in. wide, $10\frac{1}{2}$ ft. high? In the ceiling?

43. At $\$7\frac{1}{2}$ per cord, what is the value of the wood that can be put in a shed 16 ft. 8 in. long, 8 ft. 9 in. wide, 10 ft. high?

44. How many bushels of oats at $\$.66\frac{2}{3}$ a bushel will pay for $37\frac{1}{2}$ bu. of corn at $91\frac{3}{4}$ cents a bushel?

THE END.